Facoltà di Ingegneria Civile e Industriale
Ambiente e Territorio, Sicurezza

Scienza delle Costruzioni

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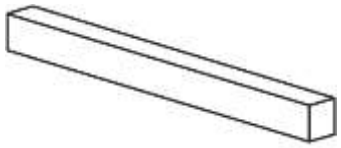
SAPIENZA
UNIVERSITÀ DI ROMA

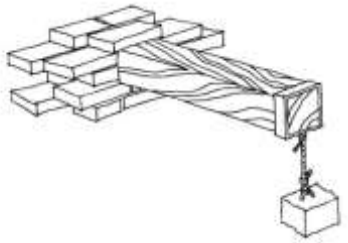
Lezione 6

Parte II - Il modello di trave elastica 1D

- Obiettivi. Definizioni. Notazioni

- Cinematica della trave
- Statica della trave
- Materiale: legame costitutivo
- Problema elastico

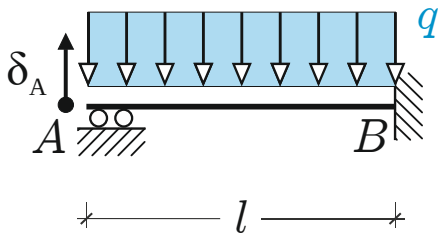


Trave elastica 1D: analisi strutturale

Obiettivi: Assegnata una trave o un sistema di travi vincolato soggetto ad azioni esterne note determinare, se esiste, la *risposta strutturale* in termini di spostamenti, deformazioni e forze interne.

esempio



Azioni Esterne → **Struttura** → **Risposta Strutturale**

- forze esterne,
- variazioni termiche,
- cedimenti vincolari,
  etc.

- geometria,
- materiali costitutivi,
- condizioni di vincolo,
  etc.

INCOGNITE

- spostamenti, deformazioni
- forze interne, tensioni

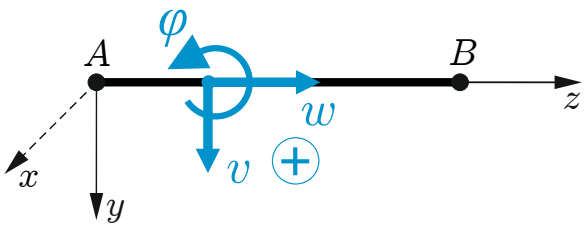
Trave elastica 1D: formulazione analitica (caso piano)

Equazioni risolventi

Incognite cinematiche

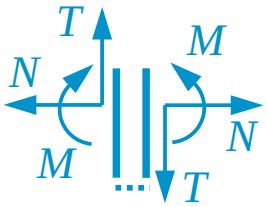
$w(z) \quad v(z) \quad \varphi(z)$

$\varepsilon(z) \quad \gamma(z) \quad \chi(z)$



Incognite statiche

$N(z) \quad T(z) \quad M(z)$



Cinematica: equazioni di congruenza

$$\begin{cases} \varepsilon(z) = w'(z) \\ \gamma(z) = \varphi(z) + v'(z) \\ \chi(z) = \varphi'(z) \end{cases} + c.c.$$

Statica: equazioni indefinite di equilibrio

$$\begin{cases} N'(z) + p(z) = 0 \\ T'(z) + q(z) = 0 \\ M'(z) - T(z) = 0 \end{cases} + c.c.$$

Materiale:

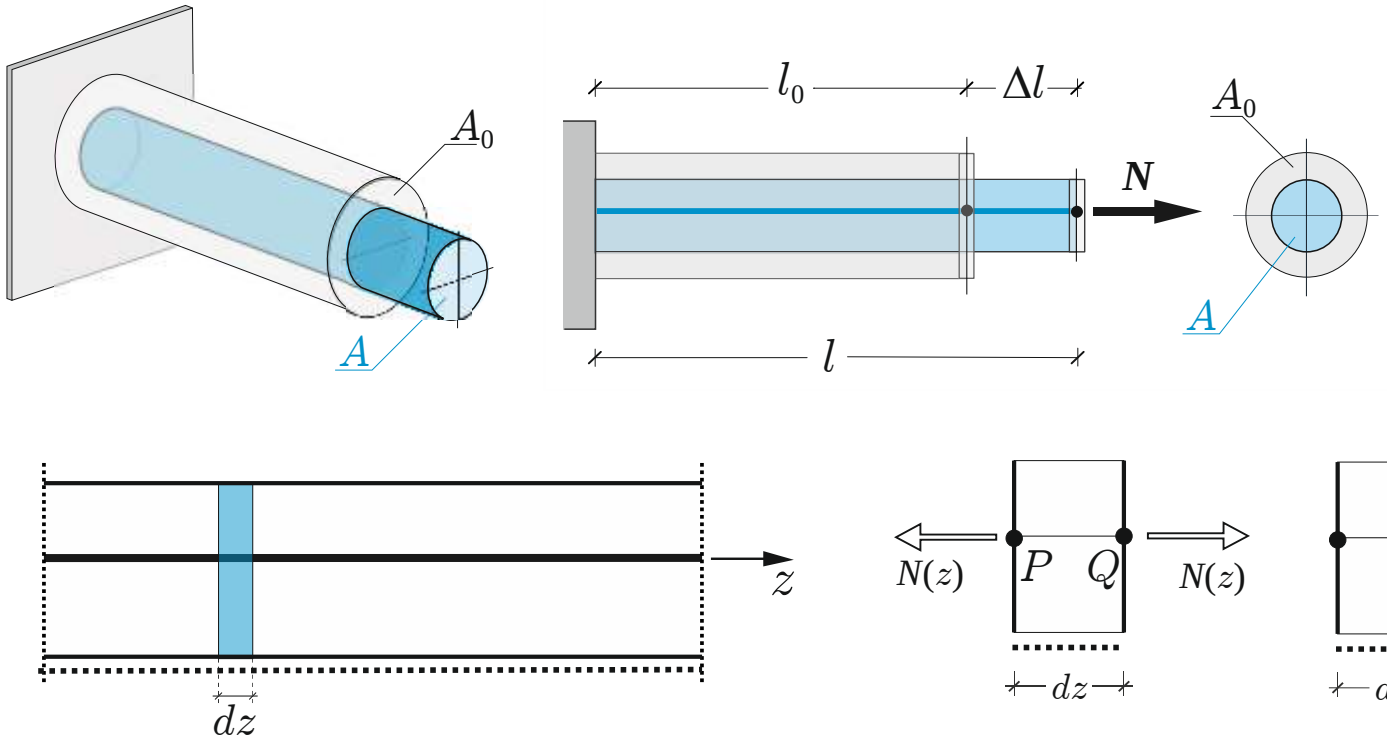
??

Parte II - Il modello di trave elastica 1D

3. Legame costitutivo

- **Obiettivi**
- **Prova uniassiale**
- **Fenomenologia**
  - materiali duttili
  - materiali fragili
- **Comportamento elastico lineare**
  - Legge di Hooke
- **Leggi di legame costitutivo per la trave**

3. Legame costitutivo: obiettivi



$N(z) \leftrightarrow \varepsilon(z)$?

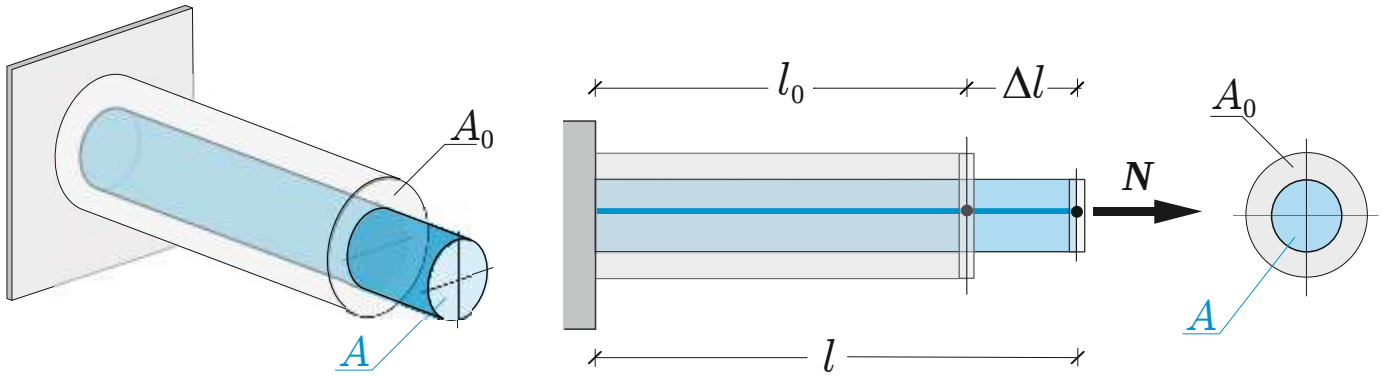
$T(z) \leftrightarrow \gamma(z)$?

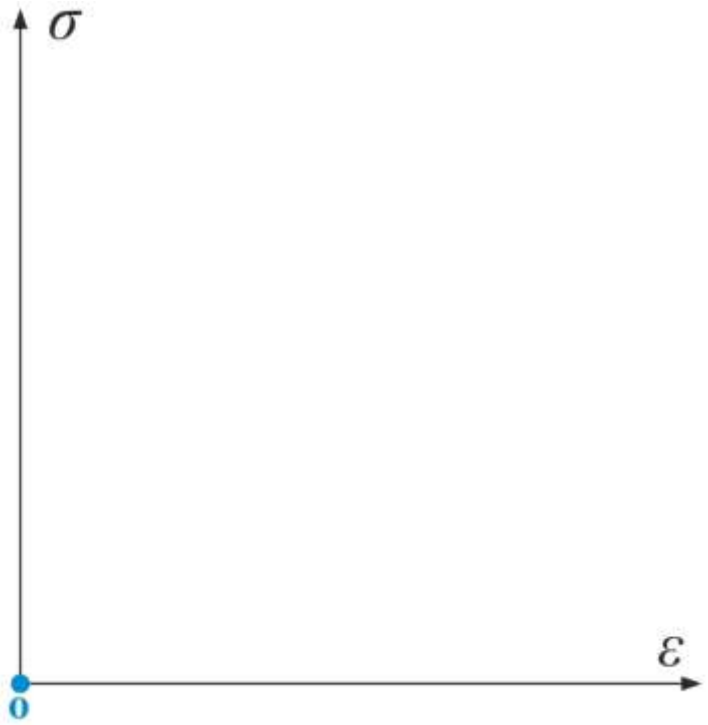
$M(z) \leftrightarrow \chi(z)$?

Materiale?

Indagini sperimentali

3. Legame costitutivo: prova uniassiale

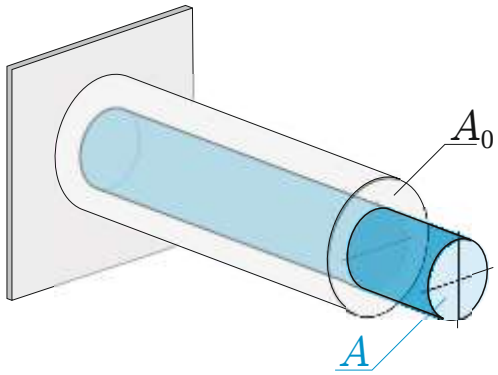


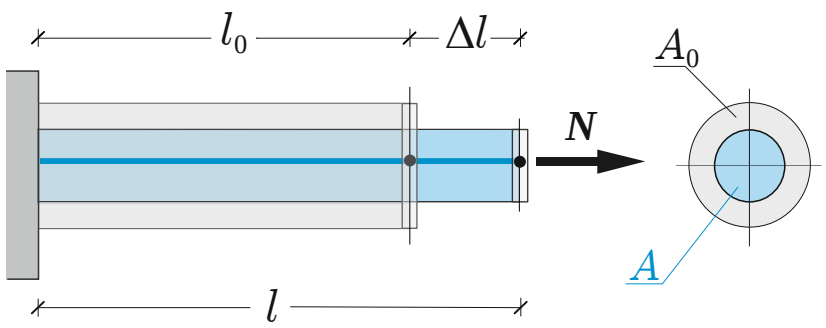


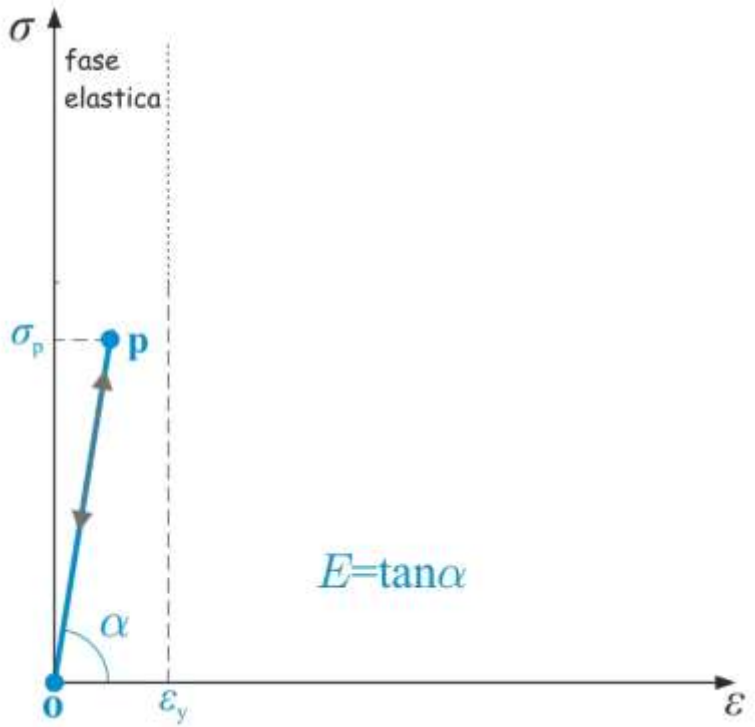
$$\sigma = \frac{N}{A_0} \cong \frac{N}{A} \qquad [FL^{-2}]$$

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0} \qquad [0]$$

3. Legame costitutivo: fenomenologia materiali duttili







FASE ELASTICA

o-p → ***comportamento elastico lineare***

o → *stato iniziale*

p → *limite di proporzionalità*

$$\sigma = \frac{N}{A_0} \cong \frac{N}{A} \qquad [FL^{-2}]$$

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0} \qquad [0]$$

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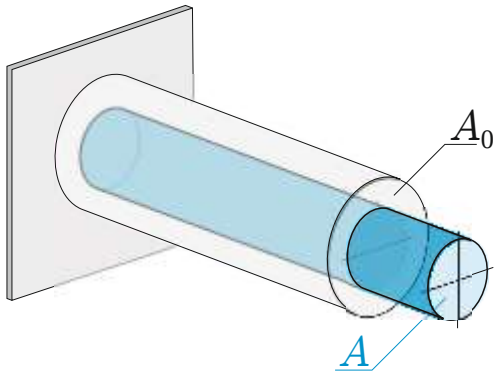
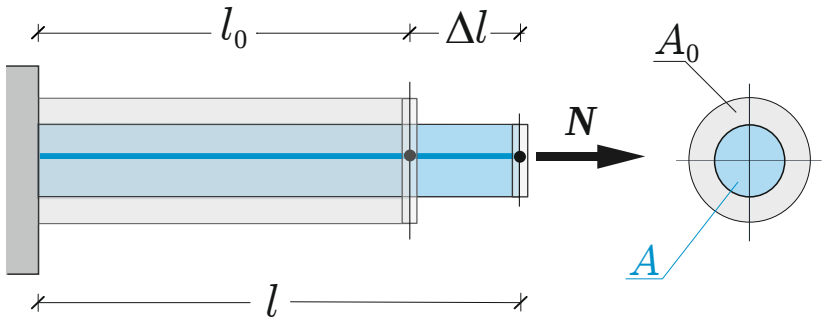
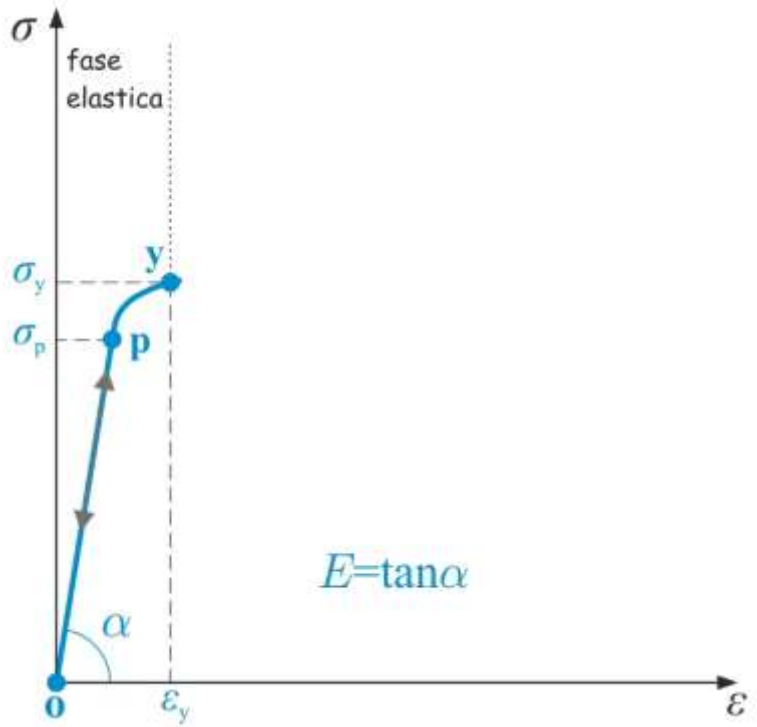






FASE ELASTICA

o-p → comportamento elastico lineare

o *→ stato iniziale*

p *→ limite di proporzionalità*

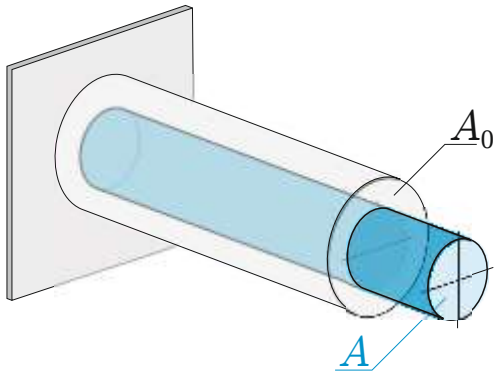
p-y → comportamento elast. non lineare

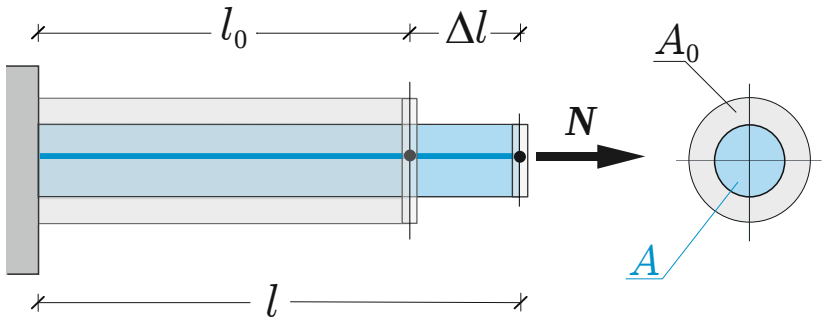
y *→ limite di elasticità*

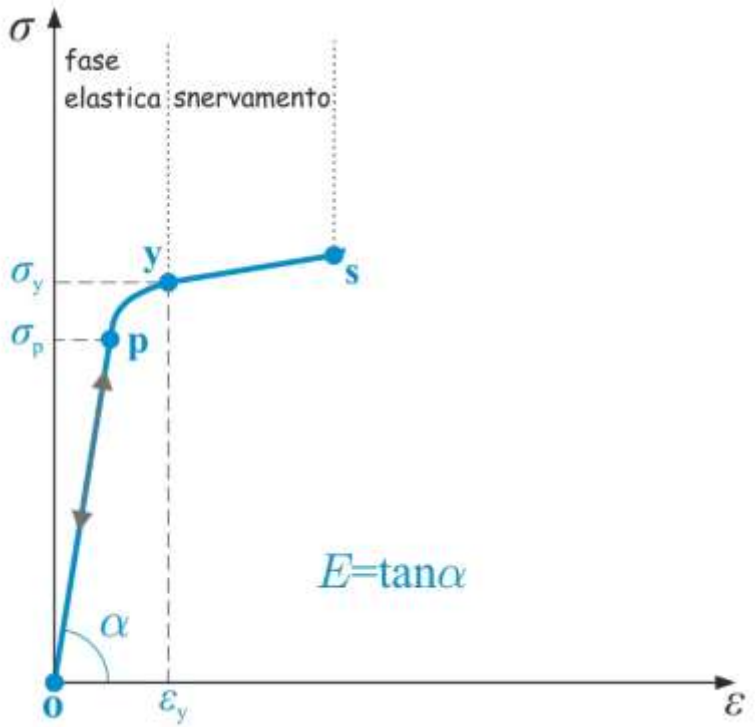
$$\sigma = \frac{N}{A_0} \cong \frac{N}{A} \qquad [FL^{-2}]$$

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0} \qquad [0]$$

3. Legame costitutivo: fenomenologia materiali duttili







FASE PLASTICA

y-s → snervamento

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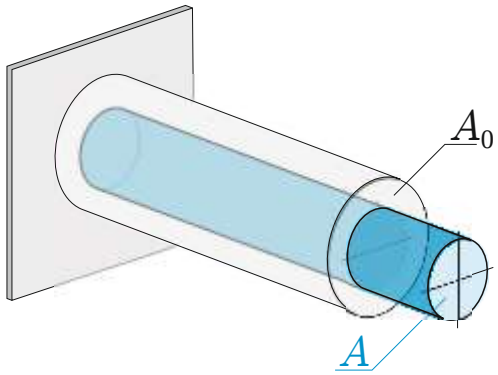
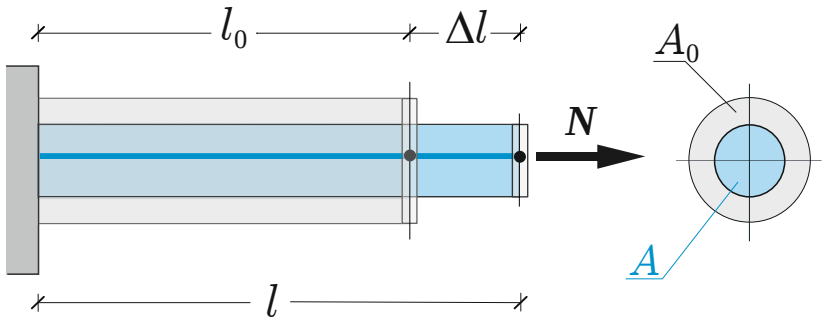
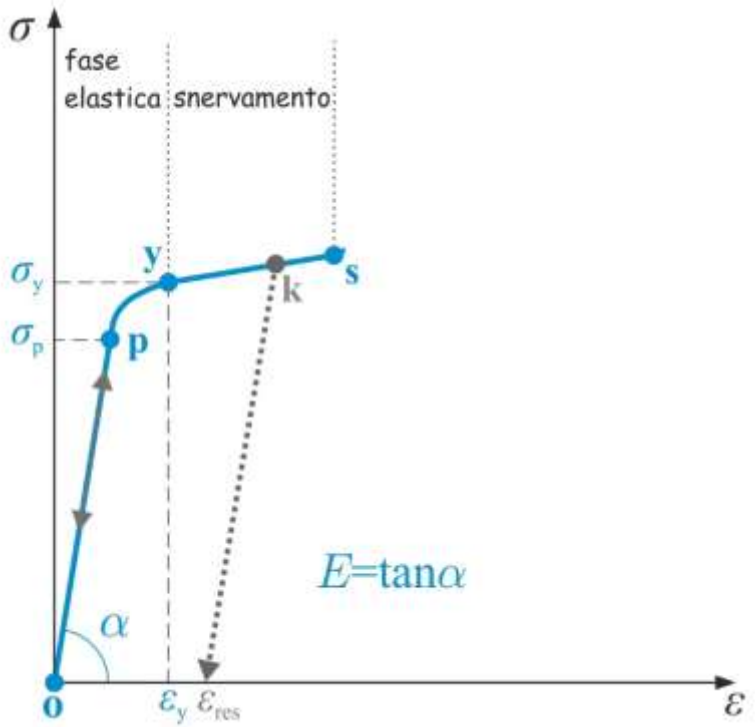
3. Legame costitutivo: fenomenologia materiali duttili







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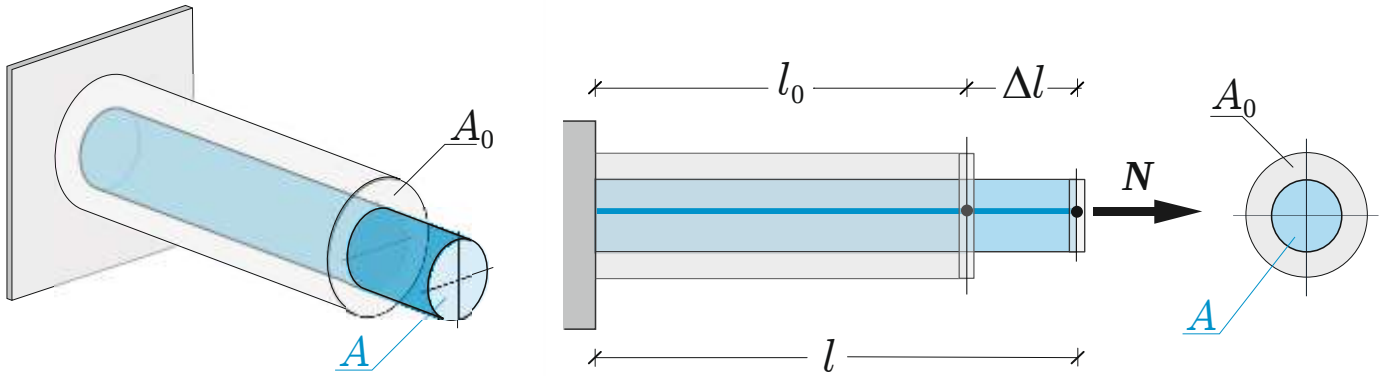
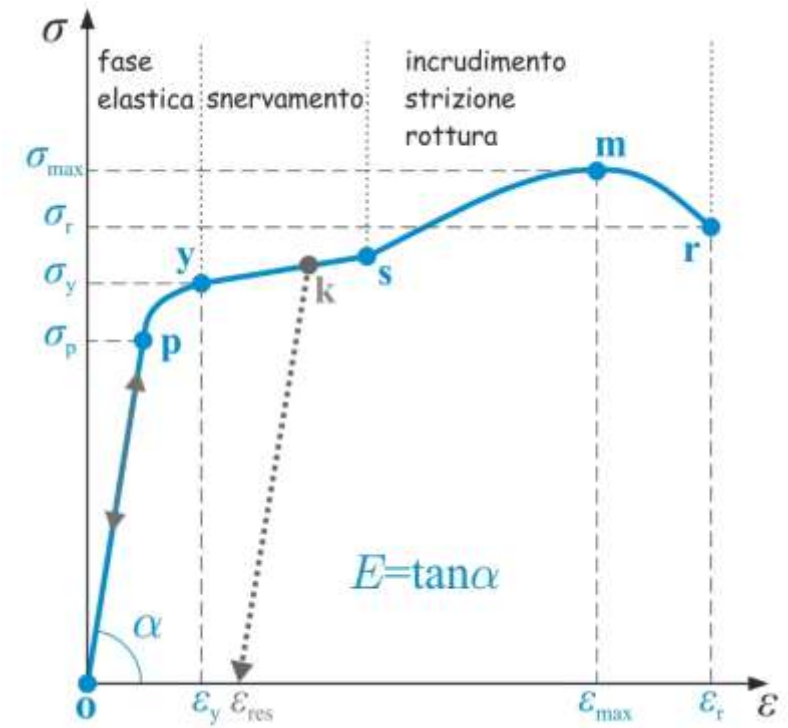
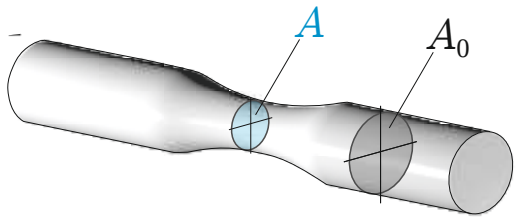




FASE PLASTICA

s-m → incrudimento

r *→ punto di rottura*



$A > A_0$

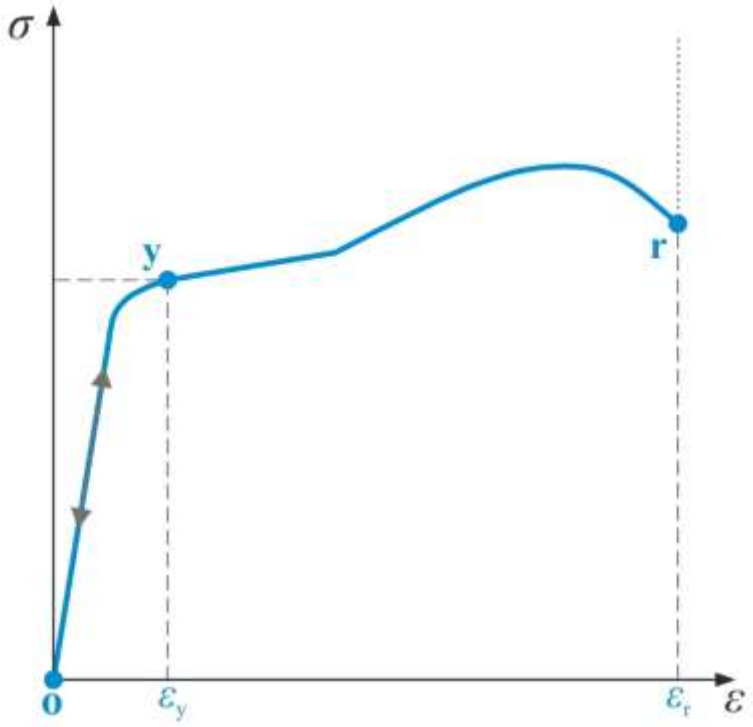
$$\sigma = \frac{N}{A_0} < \frac{N}{A} \qquad [FL^{-2}]$$

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0} \qquad [0]$$

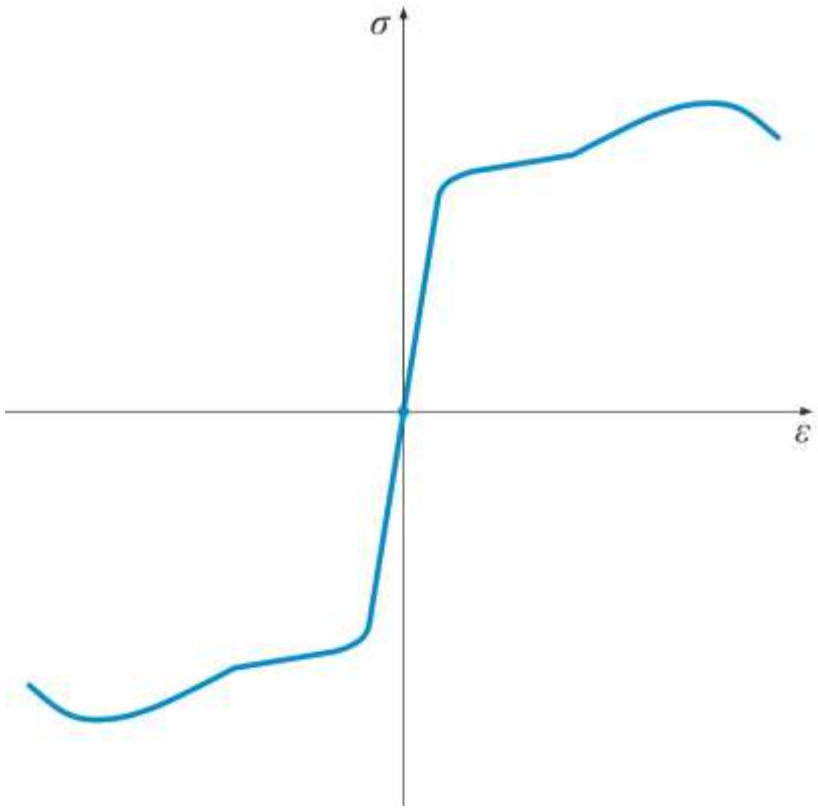
3. Legame costitutivo: fenomenologia materiali duttili

DUTTILITA'

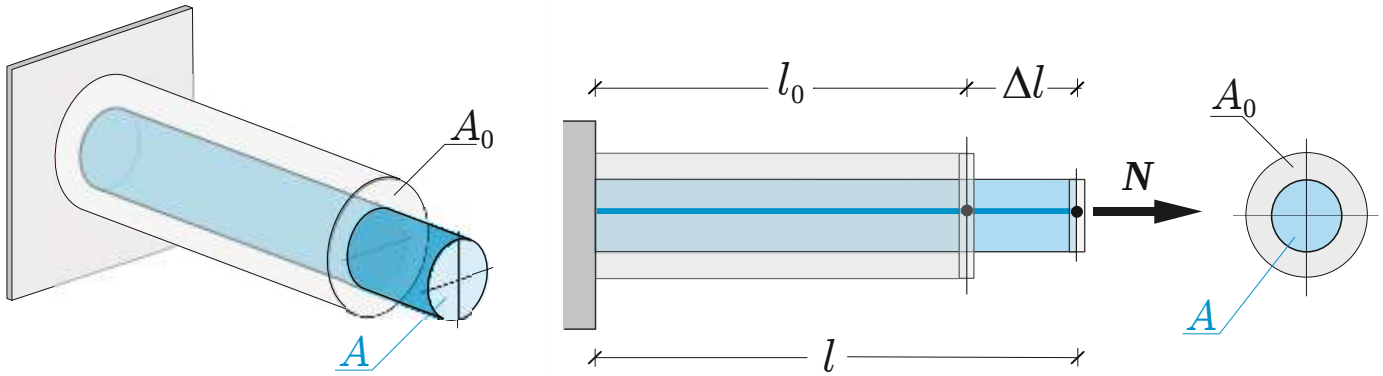
$$\mu = \varepsilon_r - \varepsilon_y$$

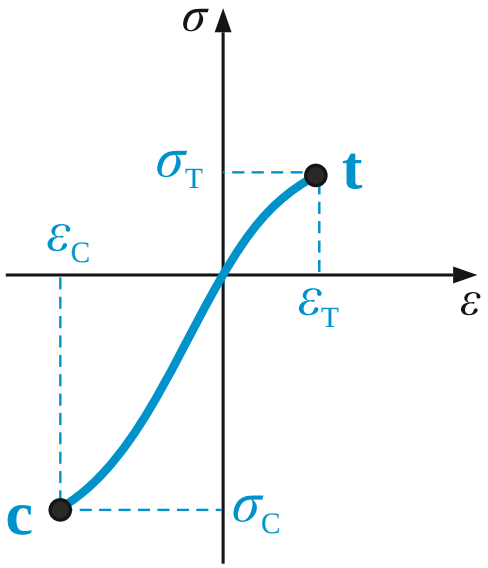


SIMMETRIA



3. Legame costitutivo: fenomenologia materiali fragili

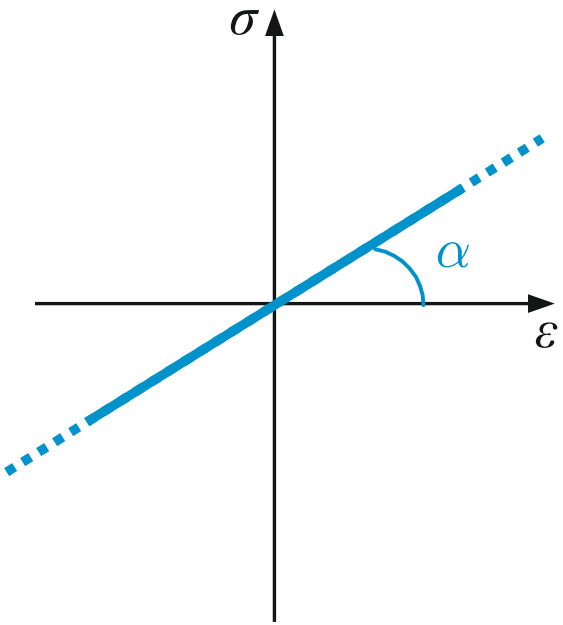




t → *punto di rottura a trazione*

c → *punto di rottura a compressione*

3. Legame costitutivo: legame elastico lineare



$$\sigma = m\,\varepsilon = \tan\alpha\;\varepsilon$$

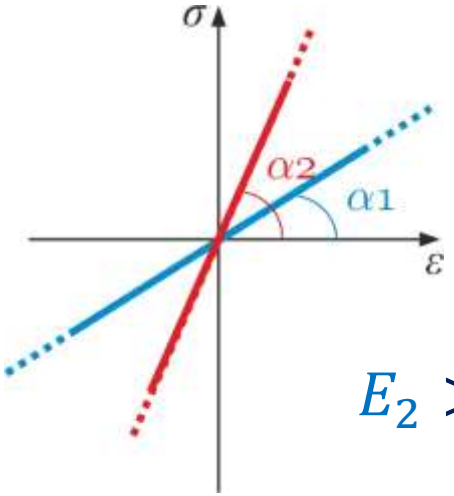
(equazione cartesiana della retta passante per l'origine di coefficiente angolare m)

$$E = \tan\alpha \quad \rightarrow \text{ Modulo di Young } [FL^{-2}]$$

$$\sigma = E\varepsilon$$

$$\varepsilon = \frac{\sigma}{E}$$

$\rightarrow$ Legge di Hooke



$$E_2 > E_1$$

3. Legame costitutivo: legame elastico lineare

Materiale	ϱ [kg/m^3]	E [GPa]	ν	α [°C^{-1}]
Acciaio strutturale	7860	200–210	0.3	$1.17 \cdot 10^{-5}$
Leghe di Alluminio	2600–2800	70–75	0.34	$2.35 \cdot 10^{-5}$
Rame	8900	120	0.36	$1.69 \cdot 10^{-5}$
Titanio	4700	110–120	0.34	$0.90 \cdot 10^{-5}$
Calcestruzzo	2400–2500	25–30	0.12	$0.99 \cdot 10^{-5}$
Muratura (mattoni pieni)	1800	30		$0.60 \cdot 10^{-5}$
Marmo	2700	55		$1.08 \cdot 10^{-5}$
Granito	2770	70		$0.72 \cdot 10^{-5}$
Legno (lungo le fibre)	400–720	9–15		$0.40 \cdot 10^{-5}$
Gomma	900	<1	0.49	$16.0 \cdot 10^{-5}$

3. Legame costitutivo: legge di Hooke per la trave

provino

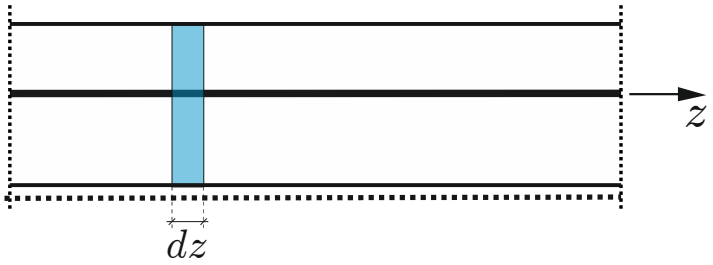
$$\varepsilon = \frac{\sigma}{E}$$

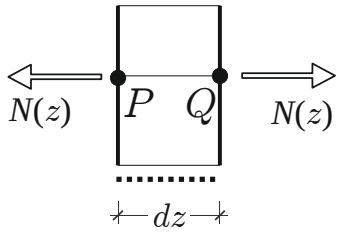
$$\sigma = \frac{N}{A}$$

$$\rightarrow \quad \varepsilon = \frac{N}{EA}$$

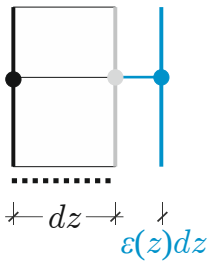
Elemento infinitesimo

$$\varepsilon(z) = \frac{N(z)}{EA}$$





forza interna



deformazione

3. Legame costitutivo: equazioni costitutive per la trave elastica

$$\varepsilon(z) = \frac{N(z)}{EA}$$

$$\gamma(z) = \frac{T(z)}{GA_t}$$

$$\chi(z) = \frac{M(z)}{EI}$$

Caratteristiche del materiale

E → Modulo di Young $[FL^{-2}]$

G → Modulo di rigidezza tangenziale $[FL^{-2}]$

Caratteristiche della sezione retta

A → Area $[L^2]$

A_t → Area di taglio $[L^2]$

$I = I_x$ → Momento d'inerzia rispetto all'asse locale x $[L^4]$

Rigidezze della trave

EA → Rigidezza assiale $[F]$

GA_t → Rigidezza di taglio $[F]$

EI → Rigidezza flessionale $[FL^2]$

$EA \to \infty$ Trave inestensibile elasticamente

$GA_t \to \infty$ Trave indeformabile a taglio (Eulero-Bernouilli)