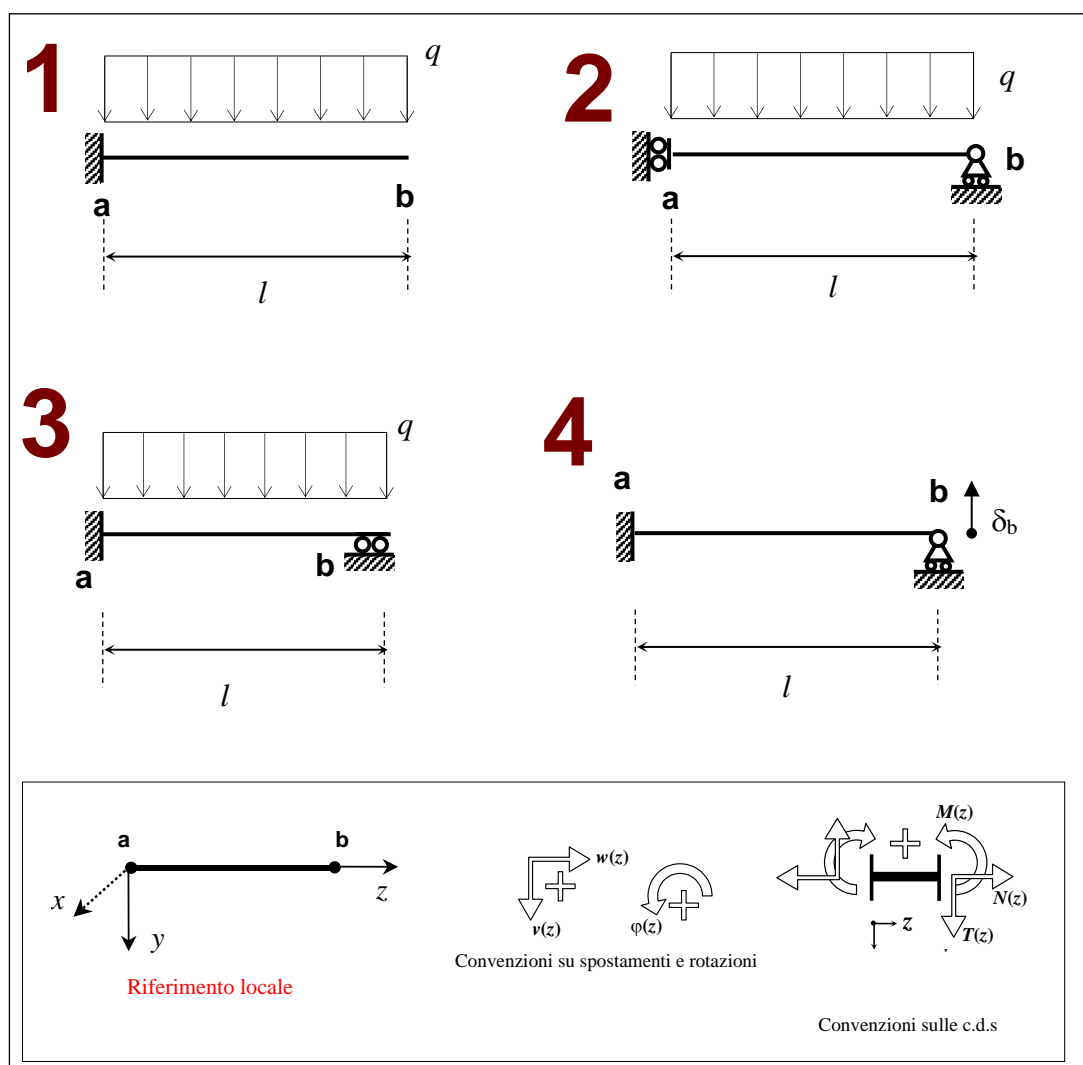


(E15a) –Linea elastica– (soluzioni sul sito)

Problemi 1-6. Per ciascuna delle travi indeformabili a taglio ($\gamma=0$) riportate in figura: **a)** scrivere le equazioni della linea elastica con le rispettive condizioni al contorno; **b)** disegnare qualitativamente la *deformata* della trave; **c)** determinare le leggi di variazione dei campi di spostamento e tensione e tracciare i relativi diagrammi. Si assumano uniformi le rigidezze con $EA=EI/l^2$. Si ricorda infine che: $N=EAw'$, $T=-EIv'''$, $M=-EIv''$.



The figure shows four beam problems (1, 2, 3, 4) and their respective diagrams and conventions.

Problem 1: A beam of length l fixed at point a and free at point b . A uniformly distributed load q acts downwards. The distance from a to b is l .

Problem 2: A beam of length l fixed at point a and supported by a roller at point b . A uniformly distributed load q acts downwards. The distance from a to b is l .

Problem 3: A beam of length l fixed at point a and supported by a roller at point b . A uniformly distributed load q acts downwards. The distance from a to b is l .

Problem 4: A beam of length l fixed at point a and supported by a roller at point b . A uniformly distributed load q acts downwards. The distance from a to b is l . The displacement at b is denoted by δ_b .

Local Reference: A coordinate system is shown with x and y axes at point a , and z axis along the beam.

Conventions on displacements and rotations: $w(z)$ (transverse displacement), $v(z)$ (rotation), and $\varphi(z)$ (rotation).

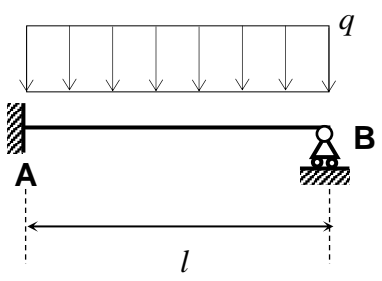
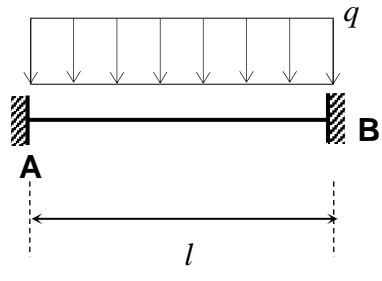
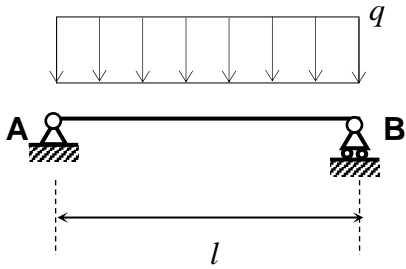
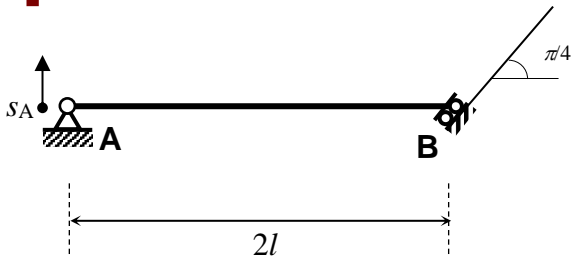
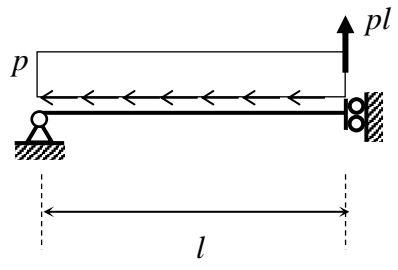
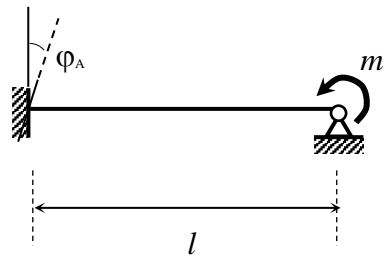
Conventions on the c.d.s (internal forces): $M(z)$ (bending moment), $N(z)$ (normal force), and $T(z)$ (shear force).

COGNOME.....
NOME.....
MAT.....

SOLUZIONI: www.pcasini.it/disg/sdc

(E15b) –Linea elastica (1 tratto regolare)

Problemi 1-6. Per ciascuna delle travi indeformabili a taglio ($\gamma=0$) riportate in figura: **a)** scrivere le equazioni della linea elastica con le rispettive condizioni al contorno; **b)** disegnare qualitativamente la *deformata* della trave; **c)** determinare le leggi di variazione dei campi di spostamento e tensione e tracciare i relativi diagrammi. Si assumano uniformi le rigidezze con $EA=EI/l^2$. Si ricorda infine che: $N=EAw'$, $T=-EIv'''$, $M=-EIv''$.

<p>1</p> 	<p>2</p> 
<p>3</p> 	<p>4</p> 
<p>5</p> 	<p>6</p>  <p>$\varphi_A = \frac{ml}{EI}$</p>

COGNOME.....
NOME.....
MAT.....

PAGINA WEB DEL CORSO:
www.pcasini.it/disg/sdc

SOLUZIONI

Scelto il sistema di riferimento locale in figura si ha rispettivamente:

Problema 1

Equazioni della linea elastica

$$\begin{cases} EA w''(z) = 0 \\ EI v''''(z) = +q \end{cases} \Rightarrow \begin{cases} w''(z) = 0 \\ v''''(z) = \frac{q}{EI} \end{cases}$$

$$\begin{array}{l} \text{Condizioni al contorno in a} \\ \text{Condizioni al contorno in b} \end{array} \begin{array}{l} \begin{cases} w(0) = 0 \\ v(0) = 0 \\ \varphi(0) = 0 \end{cases} \Rightarrow \begin{cases} w(0) = 0 \\ v(0) = 0 \\ -v'(0) = 0 \end{cases} \\ \begin{cases} N(l) = 0 \\ T(l) = 0 \\ M(l) = 0 \end{cases} \Rightarrow \begin{cases} EA w'(l) = 0 \\ -EI v''''(l) = 0 \\ -EI v'''(l) = 0 \end{cases} \end{array} \Rightarrow \begin{array}{l} \begin{cases} w(0) = 0 \\ v(0) = 0 \\ v'(0) = 0 \\ w'(l) = 0 \\ v''''(l) = 0 \\ v'''(l) = 0 \end{cases} \end{array}$$

Dalle precedenti si ricava:

$$w(z) = A_1 z + A_2; \quad A_1 = A_2 = 0$$

$$v(z) = \frac{qz^4}{24EI} + C_1 \frac{z^3}{6} + C_2 \frac{z^2}{2} + C_3 z + C_4$$

$$c_2 \rightarrow \frac{1^2 q}{2 EI}, \quad c_3 \rightarrow 0, \quad c_4 \rightarrow 0, \quad c_1 \rightarrow -\frac{1q}{EI}$$

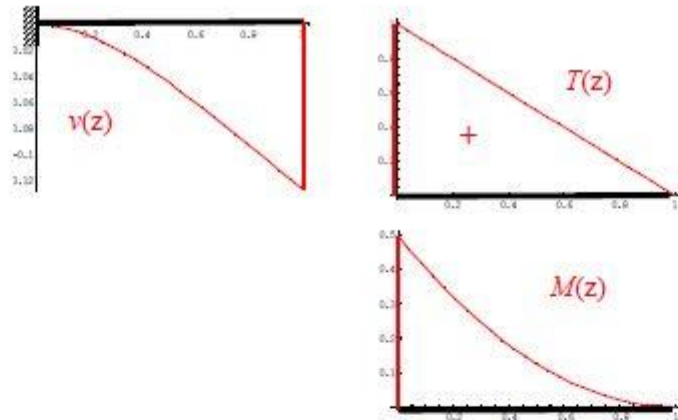
quindi:

$$v(z) = \frac{1^2 q z^2}{4 EI} - \frac{1q z^3}{6 EI} + \frac{q z^4}{24 EI}$$

$$\varphi(z) = -v'(z) = -\frac{q z (3z^2 - 3z + z^3)}{6 EI}$$

$$T(z) = -EI v''(z) = q (1 - z)$$

$$M(z) = -EI v'''(z) = -\frac{1}{2} q (1 - z)^2$$



Problema 2

Equazioni della linea elastica

$$\begin{cases} EA w''(z) = 0 \\ EI v''(z) = +q \end{cases} \Rightarrow \begin{cases} w''(z) = 0 \\ v''(z) = \frac{q}{EI} \end{cases}$$

Condizioni al contorno in a	$\begin{cases} w(0) = 0 \\ T(0) = 0 \\ \varphi(0) = 0 \end{cases} \Rightarrow \begin{cases} w(0) = 0 \\ -EIv'''(0) = 0 \\ -v'(0) = 0 \end{cases}$	$\Rightarrow \begin{cases} w(0) = 0 \\ v'''(0) = 0 \\ v'(0) = 0 \end{cases}$
Condizioni al contorno in b	$\begin{cases} v(l) = 0 \\ N(l) = 0 \\ M(l) = 0 \end{cases} \Rightarrow \begin{cases} v(l) = 0 \\ EA w'(l) = 0 \\ -EIv''(l) = 0 \end{cases}$	$\Rightarrow \begin{cases} v(l) = 0 \\ w'(l) = 0 \\ v''(l) = 0 \end{cases}$

Dalle precedenti si ricava:

$$w(z) = A_1 z + A_2; \quad A_1 = A_2 = 0$$

$$v(z) = \frac{qz^4}{24EI} + C_1 \frac{z^3}{6} + C_2 \frac{z^2}{2} + C_3 z + C_4$$

$$c_4 \rightarrow \frac{5 \cdot 1^4 q}{24 EI}, \quad c_2 \rightarrow -\frac{1^2 q}{2 EI}, \quad c_3 \rightarrow 0, \quad c_1 \rightarrow 0$$

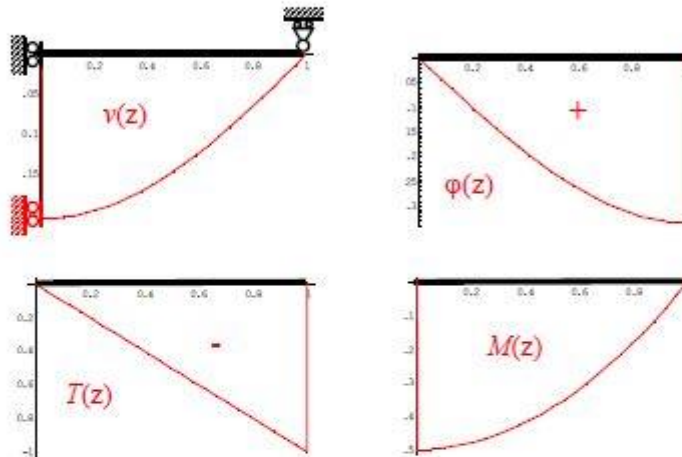
quindi:

$$v(z) = \frac{5 l^4 q}{24 EI} - \frac{l^2 q z^2}{4 EI} + \frac{q z^4}{24 EI}$$

$$\varphi(z) = -v'(z) = -\frac{q z (-3 l^2 + z^2)}{6 EI}$$

$$T(z) = -EIv'''(z) = -q z$$

$$M(z) = -EIv''(z) = \frac{1}{2} q (l^2 - z^2)$$



Problema 3

Equazioni della linea elastica

$$\begin{cases} EA w''(z) = 0 \\ EI v''(z) = +q \end{cases} \Rightarrow \begin{cases} w''(z) = 0 \\ v''(z) = \frac{q}{EI} \end{cases}$$

Condizioni al contorno in a

$$\begin{cases} w(0) = 0 \\ v(0) = 0 \\ \varphi(0) = 0 \end{cases} \Rightarrow \begin{cases} w(0) = 0 \\ v(0) = 0 \\ -v'(0) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} w(0) = 0 \\ v(0) = 0 \\ v'(0) = 0 \end{cases}$$

Condizioni al contorno in b

$$\begin{cases} N(l) = 0 \\ v(l) = 0 \\ \varphi(l) = 0 \end{cases} \Rightarrow \begin{cases} EA w'(l) = 0 \\ v(l) = 0 \\ -v'(l) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} w'(l) = 0 \\ v(l) = 0 \\ v'(l) = 0 \end{cases}$$

Dalle precedenti si ricava:

$$w(z) = A_1 z + A_2; \quad A_1 = A_2 = 0$$

$$v(z) = \frac{qz^4}{24EI} + C_1 \frac{z^3}{6} + C_2 \frac{z^2}{2} + C_3 z + C_4$$

$$c1 \rightarrow -\frac{1q}{2EI}, \quad c2 \rightarrow \frac{1^2q}{12EI}, \quad c4 \rightarrow 0, \quad c3 \rightarrow 0$$

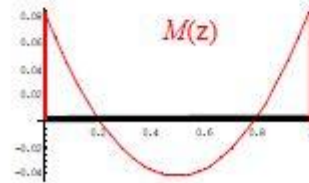
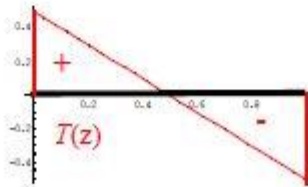
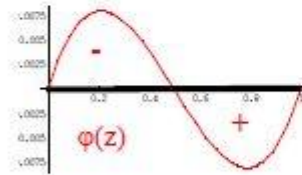
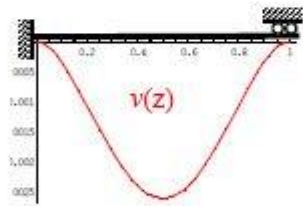
quindi:

$$v(z) = \frac{1^2q z^2}{24EI} - \frac{1q z^3}{12EI} - \frac{q z^4}{24EI}$$

$$\varphi(z) = -v'(z) = -\frac{q z (1^2 - 3 \cdot 1 z + 2 z^2)}{12EI}$$

$$T(z) = -EIv''(z) = \frac{1}{2} q (1 - 2z)$$

$$M(z) = -EIv'''(z) = -\frac{1}{12} q (1^2 - 6 \cdot 1 z + 6 z^2)$$



Problema 4

Equazioni della linea elastica

$$\begin{cases} EA w''(z) = 0 \\ EI v''''(z) = 0 \end{cases} \Rightarrow \begin{cases} w''(z) = 0 \\ v''''(z) = 0 \end{cases}$$

Condizioni al contorno in a

$$\begin{cases} w(0) = 0 \\ v(0) = 0 \\ \varphi(0) = 0 \end{cases} \Rightarrow \begin{cases} w(0) = 0 \\ v(0) = 0 \\ -v'(0) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} w(0) = 0 \\ v(0) = 0 \\ v'(0) = 0 \end{cases}$$

Condizioni al contorno in b

$$\begin{cases} v(l) = -\delta_b \\ N(l) = 0 \\ M(l) = 0 \end{cases} \Rightarrow \begin{cases} v(l) = -\delta_b \\ EA w'(l) = 0 \\ -EI v''(l) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} v(l) = -\delta_b \\ w'(l) = 0 \\ v''(l) = 0 \end{cases}$$

Dalle precedenti si ricava:

$$w(z) = A_1 z + A_2; A_1 = A_2 = 0$$

$$v(z) = C_1 \frac{z^3}{6} + C_2 \frac{z^2}{2} + C_3 z + C_4$$

$$c1 \rightarrow \frac{3\delta_b}{1^3}, c2 \rightarrow -\frac{3\delta_b}{1^2}, c3 \rightarrow 0, c4 \rightarrow 0$$

quindi:

$$v(z) = -\frac{3z^2\delta_b}{2 \cdot 1^2} + \frac{z^3\delta_b}{2 \cdot 1^3}$$

$$\varphi(z) = -v'(z) = \frac{3(2 \cdot 1 - z)z\delta_b}{2 \cdot 1^3}$$

$$T(z) = -EIv'''(z) = -\frac{3EI\delta_b}{1^3}$$

$$M(z) = -EIv''(z) = \frac{3EI(1-z)\delta_b}{1^3}$$

