SISTEMI DI TRAVI ISOSTATICI

ESERCITAZIONI

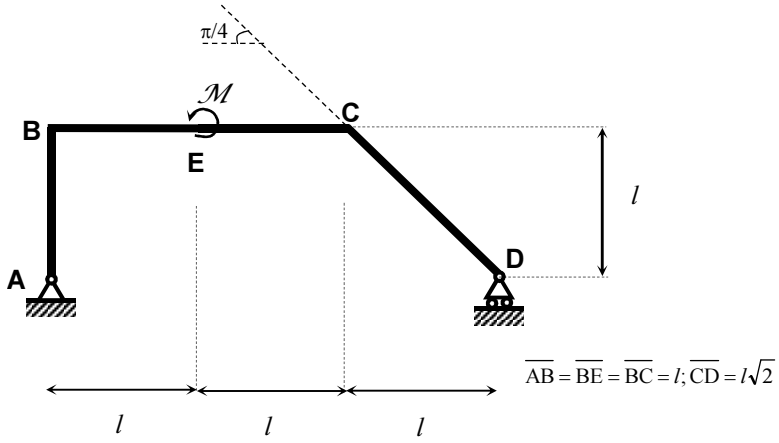
a cura di P. Casini

Testi consigliati:

1. P. Casini, M. Vasta, *Scienza delle Costruzioni*, CittàStudi (UTET università), 2011.
2. C. Comi, L. Corradi Dell'Acqua: *Introduzione alla meccanica strutturale*, McGraw-Hill, 2007.
3. E. Viola: *Complementi ed esercizi di Scienza delle Costruzioni*, Voll. 1, 2, Pitagora

1. Sistemi staticamente determinati: Esercizi svolti

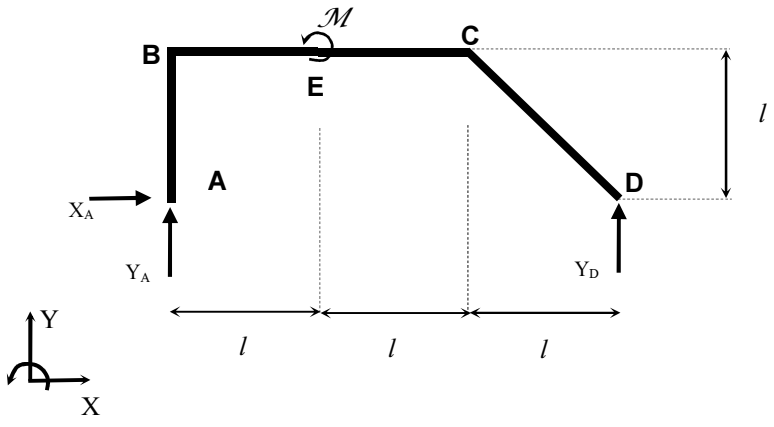
Esercizio 1.1



Con riferimento al sistema isostatico mostrato figura si chiede di:

- Calcolare le reazioni vincolari.
- Tracciare i diagrammi delle Caratteristiche della Sollecitazione.

• Calcolo delle reazioni vincolari



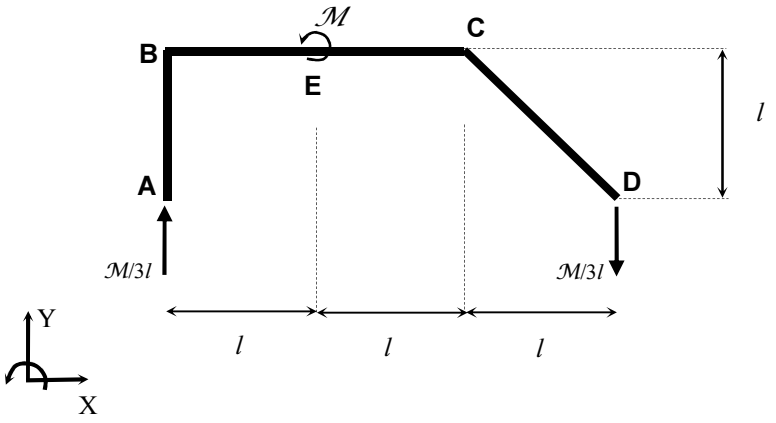
$\Sigma X = 0 \qquad \Rightarrow \qquad X_A = 0$

$\Sigma Y = 0 \quad \Rightarrow \quad Y_A + Y_D = 0$

$\Sigma M_A = 0 \quad \Rightarrow \quad \mathcal{M} + 3l Y_D = 0 \quad \Rightarrow \quad Y_D = -\mathcal{M}/3l$

$\Rightarrow \quad Y_A = \mathcal{M}/3l$

- Riepilogo delle reazioni vincolari

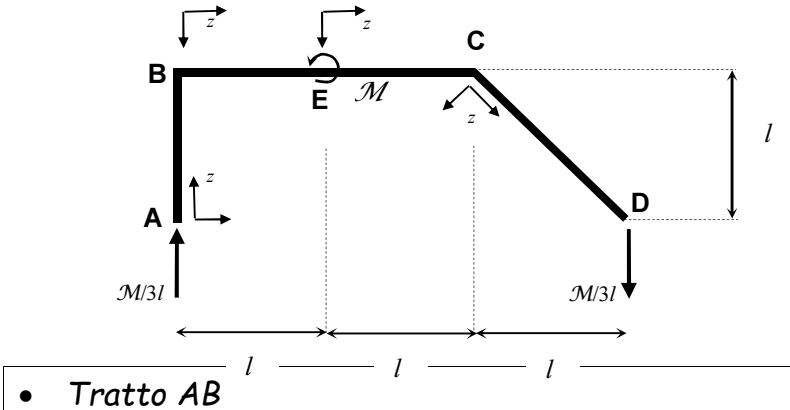


- ***Verifiche***

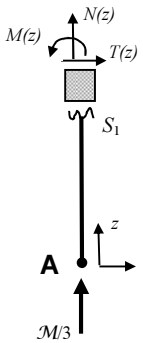
$\Sigma Y = 0 \quad \Rightarrow \quad \mathcal{M}/3l - \mathcal{M}/3l = 0$ ✓

$\Sigma M_E = 0 \quad \Rightarrow \quad \mathcal{M} - (\mathcal{M}/3l)l - (\mathcal{M}/3l)(2\,l) = 0$ ✓

- Caratteristiche della Sollecitazione)



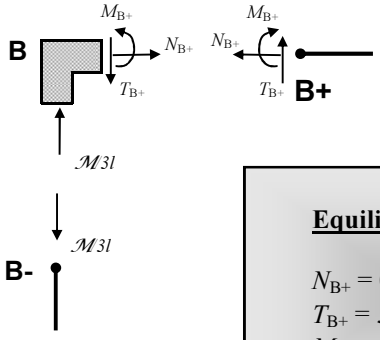
- *Tratto AB*



$N(z) + \mathcal{M}/3l = 0$ $\quad N(z) = -\mathcal{M}/3l$
$T(z) = 0$ $\quad T(z) = 0$
$\Sigma M_{S1} = 0$ $\quad M(z) = 0$

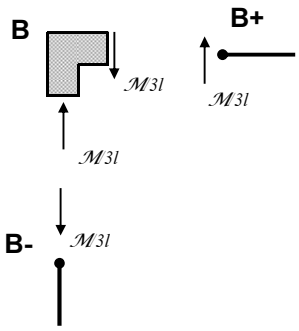
- $z = 0 \Rightarrow S_1 \equiv A$ $\quad N(0) = N_A = -\mathcal{M}/3l$; $T(0) = T_A = 0$; $M(0) = M_A = 0$
- $z = l \Rightarrow S_1 \equiv B^-$ $\quad N(l) = N_{B-} = -\mathcal{M}/3l$; $T(l) = T_{B-} = 0$; $M(l) = M_{B-} = 0$

• *Nodo B*

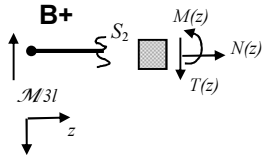


Equilibrio del Nodo B

$N_{B+} = 0$
$T_{B+} = \mathcal{M}/3l$
$M_{B+} = 0$



• *Tratto BE*



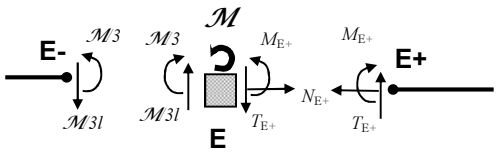
$N(z) = 0$ $\quad N(z) = 0$
$T(z) - \mathcal{M}/3l = 0$ $\quad T(z) = \mathcal{M}/3l$
$\Sigma M_{S1} = 0$ $\quad M(z) = (\mathcal{M}/3l)z$

- $z = 0 \Rightarrow S_2 \equiv B+$ $\quad N(0) = N_{B+} = 0$

$T(0) = T_{B+} = \mathcal{M}/3l$
$M(0) = M_{B+} = 0$

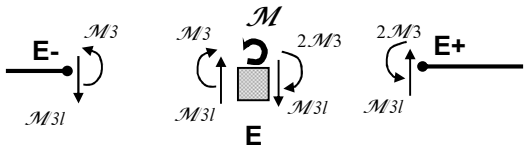
- $z = l \Rightarrow S_2 \equiv$ E- $\quad N(l) = N_{E-} = 0$
  $T(l) = T_{E-} = \mathcal{M}/3l$
  $M(l) = M_{E-} = \mathcal{M}/3$

• *Nodo E*

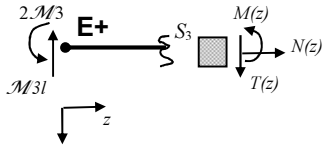


Equilibrio del Nodo E

$N_{E+} = 0$
$T_{E+} = \mathcal{M}/3l$
$M_{E+} = -(2/3)\mathcal{M}$



• *Tratto EC*



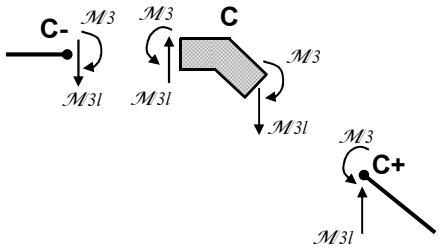
$N(z) = 0$ $\quad N(z) = 0$
$T(z) - \mathcal{M}/3l = 0$ $\quad T(z) = \mathcal{M}/3l$
$\Sigma M_{S3} = 0$ $\quad M(z) = -(2/3)\mathcal{M} + (\mathcal{M}/3l)z$

- $z = 0 \Rightarrow S_3 \equiv$ E+ $\quad N(0) = N_{E+} = 0$
  $T(0) = T_{E+} = \mathcal{M}/3l$

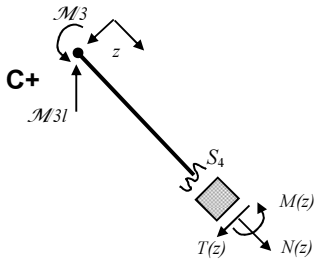
$M(0) = M_{E+} = -(2/3)\mathcal{M}$

- $z = l \Rightarrow S_3 \equiv$ C- $\quad N(l) = N_{C-} = 0$
  $T(l) = T_{C-} = \mathcal{M}/3l$
  $M(l) = M_{C-} = -\mathcal{M}/3$

• Nodo C



• Tratto CD



$N(z) - \frac{\mathcal{M}}{3l}\frac{\sqrt{2}}{2} = 0 \qquad N(z) = \frac{\mathcal{M}}{6l}\sqrt{2}$

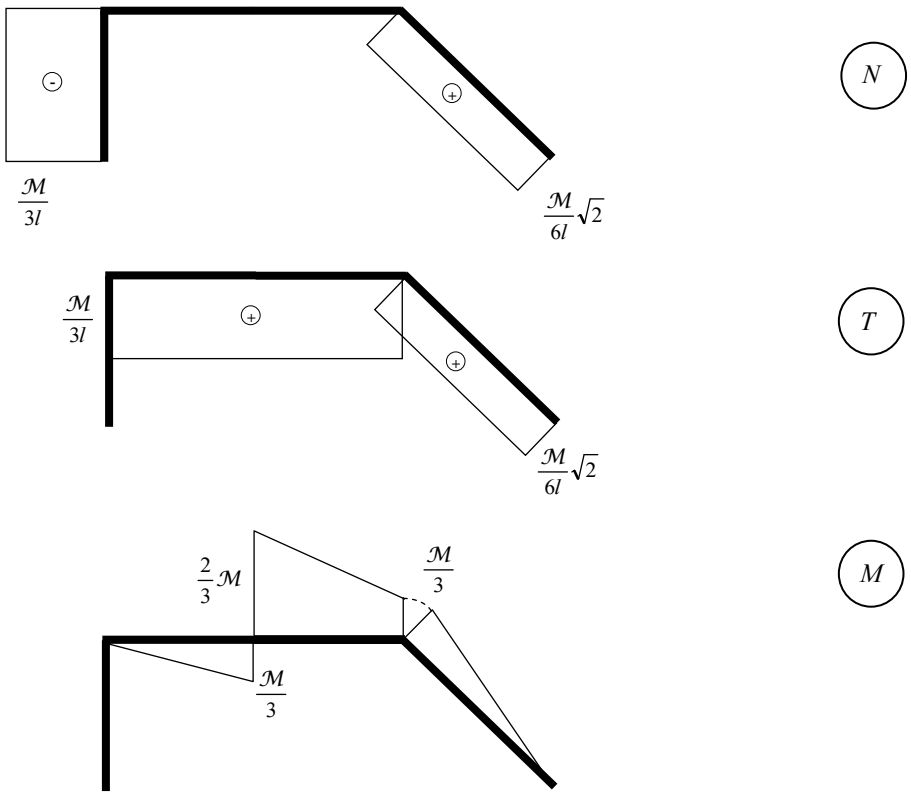
$T(z) - \frac{\mathcal{M}}{3l}\frac{\sqrt{2}}{2} = 0 \qquad T(z) = \frac{\mathcal{M}}{6l}\sqrt{2}$

$\Sigma M_{S4} = 0 \qquad M(z) = -\frac{\mathcal{M}}{3} + \frac{\mathcal{M}\sqrt{2}}{6l}z$

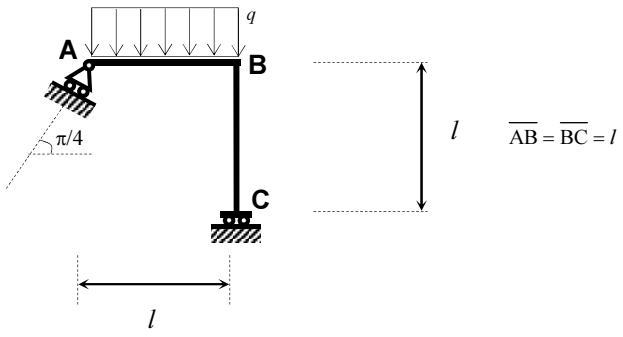
- $z = 0 \Rightarrow S_4 \equiv$ C+ $\quad N(0) = N_{C+} = \frac{\mathcal{M}}{6l}\sqrt{2}$
  $T(0) = T_{C+} = \frac{\mathcal{M}}{6l}\sqrt{2}$
  $M(0) = M_{C+} = -\mathcal{M}/3$

- $z = l\sqrt{2} \Rightarrow S_4 \equiv$ D $\quad N(l\sqrt{2}) = N_D = \frac{\mathcal{M}}{6l}\sqrt{2}$
  $T(l\sqrt{2}) = T_D = \frac{\mathcal{M}}{6l}\sqrt{2}$
  $M(l\sqrt{2}) = M_D = 0$

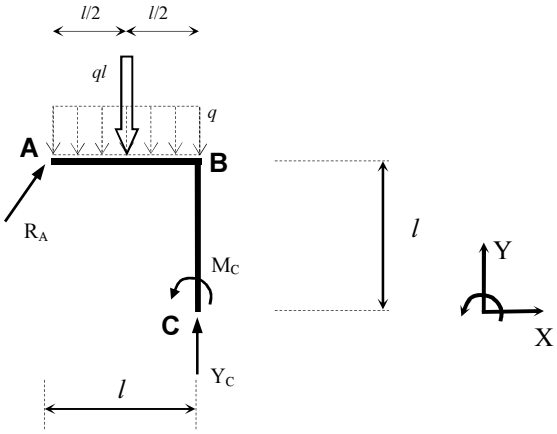
- Diagrammi delle Caratteristiche della Sollecitazione



Esercizio 1.2



- Calcolo delle reazioni vincolari

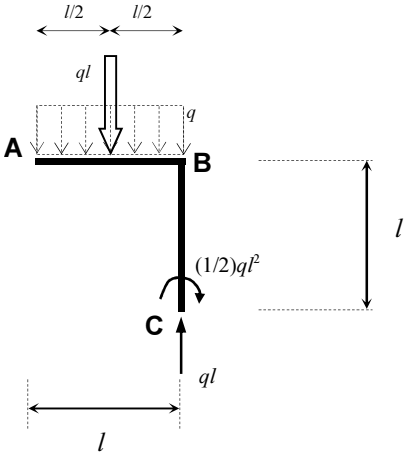


$\Sigma X = 0 \quad \Rightarrow \quad R_A \frac{\sqrt{2}}{2} = 0 \quad \Rightarrow \quad R_A = 0$

$\Sigma Y = 0 \quad \Rightarrow \quad Y_C - ql = 0 \quad \Rightarrow \quad Y_C = ql$

$\Sigma M_C = 0 \quad \Rightarrow \quad (ql)(l/2) + M_C = 0 \quad \Rightarrow \quad M_C = -\frac{ql^2}{2}$

- Riepilogo delle reazioni vincolari

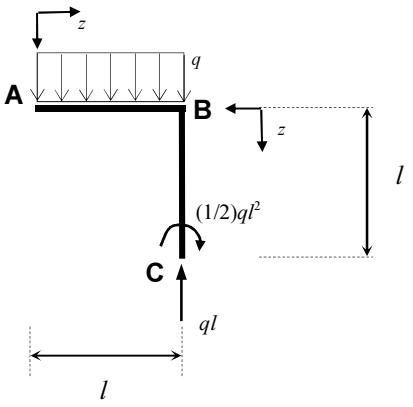


- ***Verifiche***

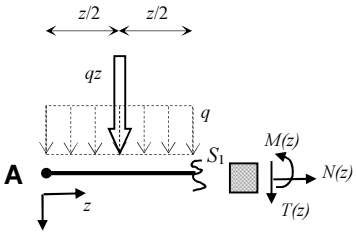
$\Sigma Y = -ql + ql = 0$ ✓

$\Sigma M_A = -ql\frac{l}{2} + ql^2 - \frac{ql^2}{2} = 0$ ✓

- Caratteristiche della Sollecitazione



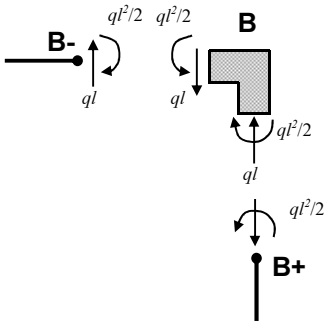
- *Tratto AB*



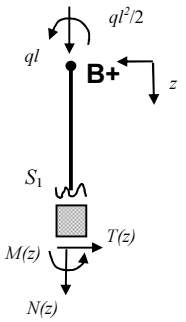
$N(z) = 0$
$T(z) = -qz$
$M(z) = -(1/2)qz^2$

- $z = 0 \Rightarrow S_1 \equiv A$ — $N(0) = N_A = 0$; $T(0) = T_A = 0$; $M(0) = M_A = 0$
- $z = l \Rightarrow S_1 \equiv B^-$ — $N(l) = N_{B^-} = 0$; $T(l) = T_{B^-} = -ql$; $M(l) = M_{B^-} = -(1/2)ql^2$

- *Nodo B*

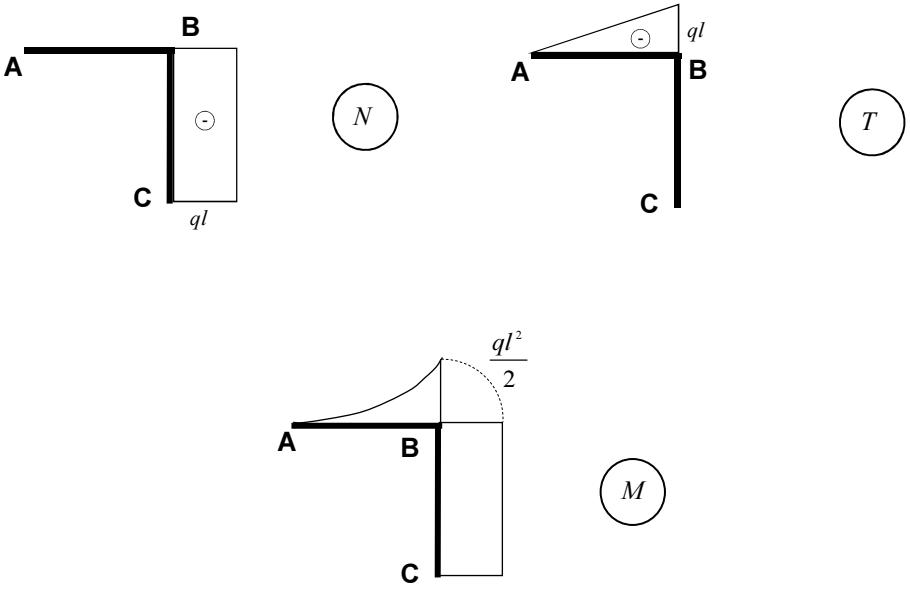


- *Tratto BC*

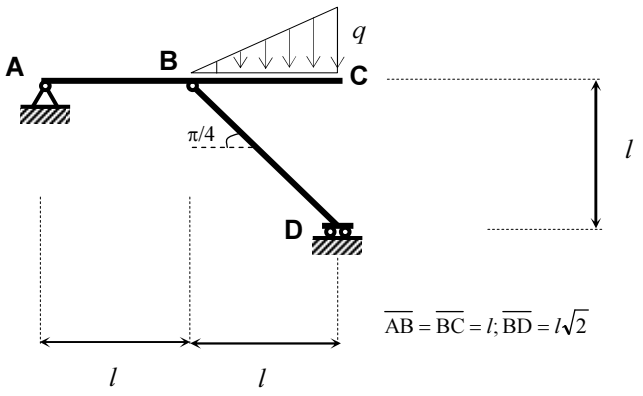


$N(z) = -\,ql$
$T(z) = 0$
$M(z) = -\,(1/2)ql^2$

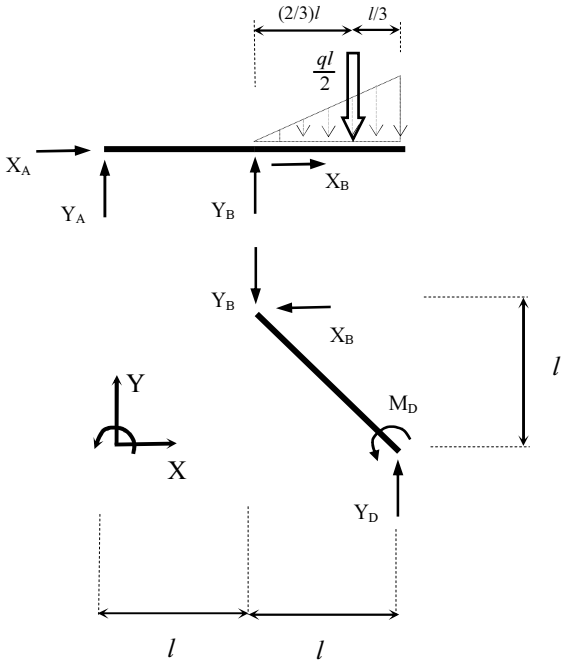
- Diagrammi delle Caratteristiche della Sollecitazione



Esercizio 1.3



- Calcolo delle reazioni vincolari



- **Equilibrio asta BD**

$-X_B = 0$
$Y_D - Y_B = 0$
$M_D + X_B l + Y_B l = 0$

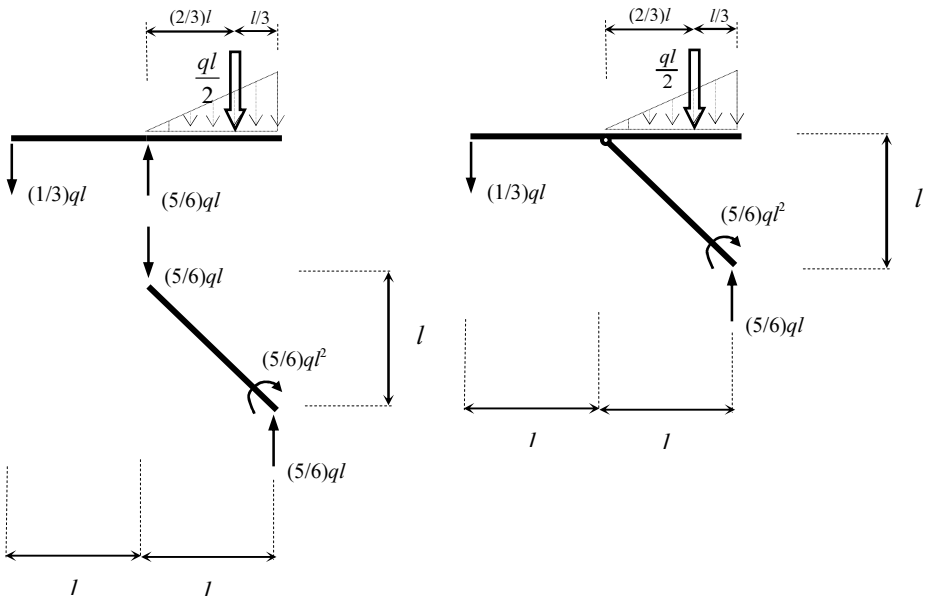
$X_B = 0$
$Y_B = Y_D$
$M_D = -Y_B l$

- **Equilibrio asta ABC**

$X_A = 0$
$Y_A + Y_B - ql/2 = 0$
$Y_B l - (ql/2)[l + (2/3)l] = 0$

$X_A = 0$ $\quad X_B = 0$
$Y_B = (5/6)ql \quad Y_A = -ql/3$
$Y_D = (5/6)ql$
$M_D = -(5/6)ql^2$

- Riepilogo delle reazioni vincolari

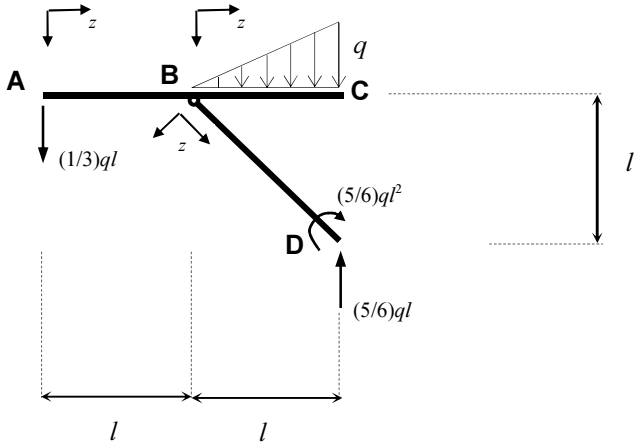


- ***Verifiche***

$\Sigma Y = 0 \quad \Rightarrow \quad -\frac{ql}{3}+\frac{5ql}{6}-\frac{ql}{2}=0$ ✓

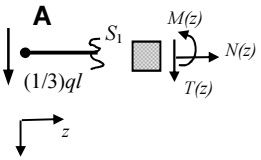
$\Sigma M_B = 0 \quad \Rightarrow \quad \frac{ql}{3}l+\frac{5ql}{6}l-\frac{5}{6}ql^2-\frac{ql}{2}\frac{2}{3}l=0$ ✓

- Caratteristiche della Sollecitazione



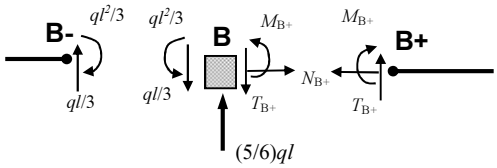
- *Tratto AB*

$N(z) = 0$
$T(z) = -(1/3)ql$
$M(z) = -(1/3)ql\, z$



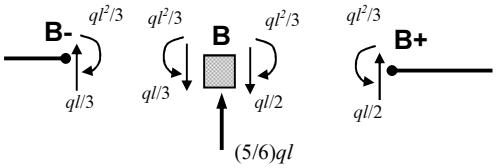
- $z = 0 \Rightarrow S_1 \equiv A$: $N(0) = N_A = 0$, $T(0) = T_A = -(1/3)ql$, $M(0) = M_A = 0$
- $z = l \Rightarrow S_1 \equiv B^-$: $N(l) = N_{B-} = 0$, $T(l) = T_{B-} = -(1/3)ql$, $M(l) = M_{B-} = -(1/3)ql^2$

- *Nodo B*

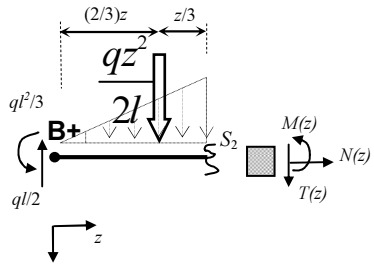


Equilibrio del Nodo B

$N_{B+} = 0$
$T_{B+} = ql/2$
$M_{B+} = -ql^2/3$



- ***Tratto BC***



La legge di variazione del carico distribuito nel sistema di riferimento locale è $q(z) = \frac{q}{l} z$. Le equazioni di Equilibrio del tratto B^+S_2 comportano (vedi Fig.):

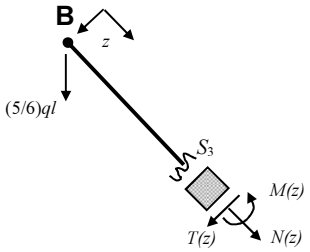
$N(z) = 0$

$T(z) = \frac{ql}{2} - \frac{qz^2}{2l}$

$M(z) = -\frac{ql^2}{3} + \frac{ql}{2} z - \frac{qz^3}{6}$

- $z = 0 \Rightarrow S_2 \equiv B+$ $\quad N(0) = N_{B+} = 0$; $T(0) = T_{B+} = (1/2)ql$; $M(0) = M_{B+} = -(1/3)ql^2$
- $z = l \Rightarrow S_2 \equiv C$ $\quad N(l) = N_C = 0$; $T(l) = T_C = 0$; $M(l) = M_C = 0$

- ***Tratto BD***



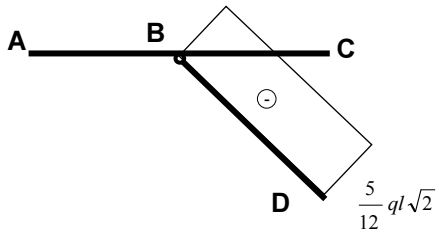
$N(z) + \frac{5}{6} ql \frac{\sqrt{2}}{2} = 0 \qquad N(z) = -\frac{5}{12} ql\sqrt{2}$

$T(z) + \frac{5}{6} ql \frac{\sqrt{2}}{2} = 0 \qquad T(z) = -\frac{5}{12} ql\sqrt{2}$

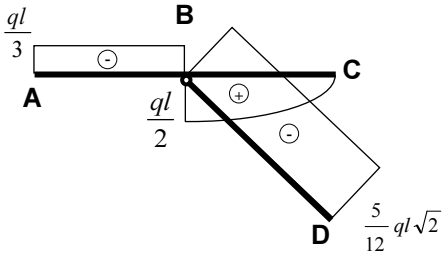
$\Sigma M_{S3} = 0 \qquad M(z) = -\frac{5}{12} ql\sqrt{2}\, z$

- $z = 0 \Rightarrow S_3 \equiv B$ $\quad N(0) = N_B = -\frac{5}{12} ql\sqrt{2}$; $T(0) = T_B = -\frac{5}{12} ql\sqrt{2}$; $M(0) = M_B = 0$
- $z = l\sqrt{2} \Rightarrow S_3 \equiv D$ $\quad N(l\sqrt{2}) = N_D = -\frac{5}{12} ql\sqrt{2}$; $T(l\sqrt{2}) = T_D = -\frac{5}{12} ql\sqrt{2}$; $M(l\sqrt{2}) = M_D = -\frac{5}{6} ql^2$

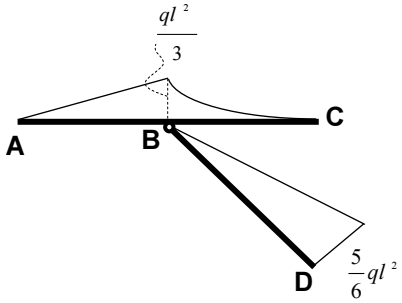
- Diagrammi delle Caratteristiche della Sollecitazione



N

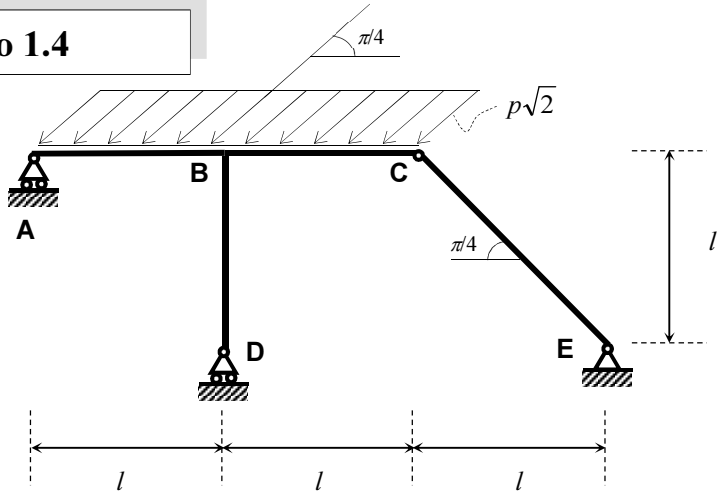


T



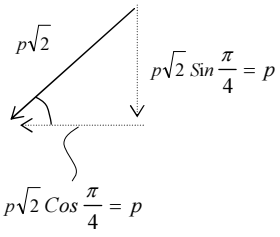
M

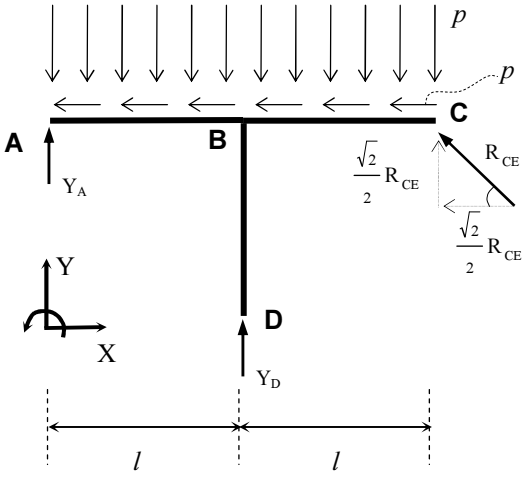
Esercizio 1.4



- Calcolo delle reazioni vincolari

Il sistema assegnato è staticamente determinato (la dimostrazione si lascia per esercizio). Come prima cosa conviene scomporre il carico distribuito in una componente perpendicolare e in una componente parallela alla direzione AC. Si osserva poi che l'asta EC è sottoposta unicamente a forza normale; pertanto le reazioni vincolari delle cerniere C ed E devono essere parallele alla direzione EC.





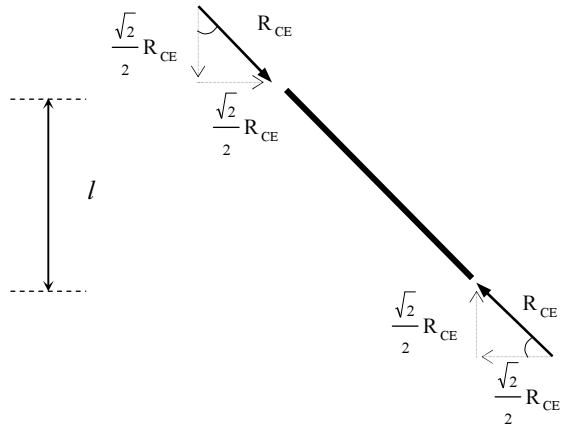
- **Equilibrio corpo ABCD**

$$-\int_0^{2l} p\,dx - \frac{\sqrt{2}}{2}R_{CE} = 0$$

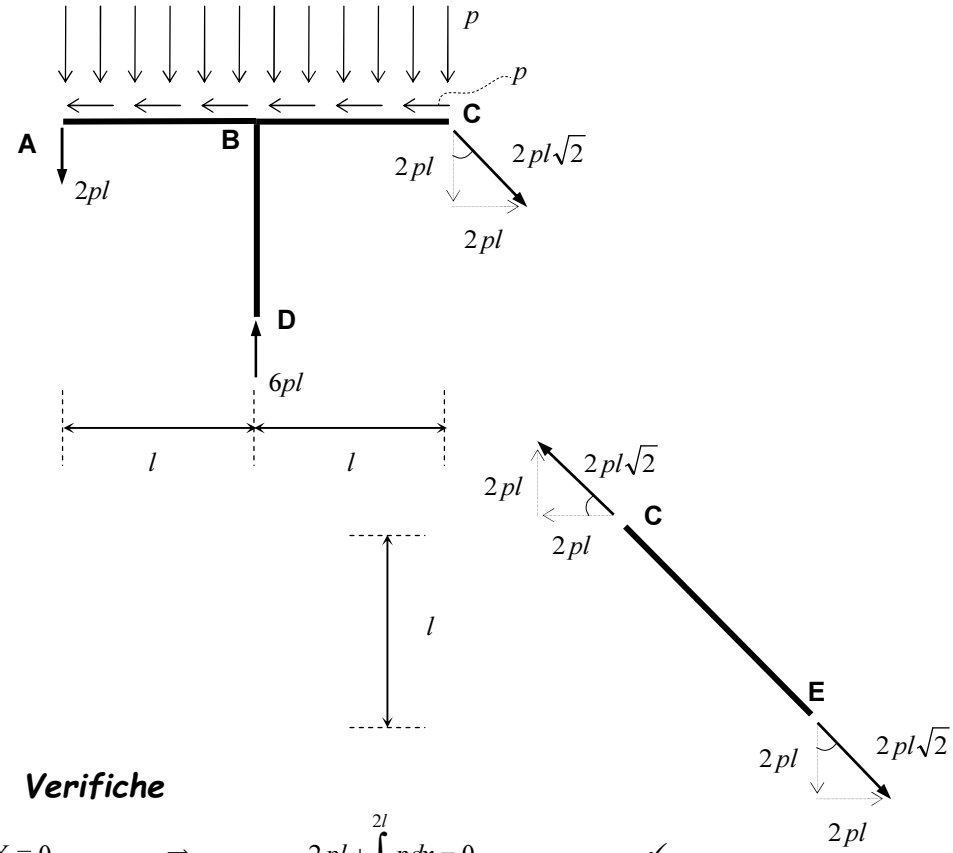
$$-\int_0^{2l} p\,dx + \frac{\sqrt{2}}{2}R_{CE} + Y_A + Y_D = 0$$

$$-\int_0^{2l} px\,dx + \frac{\sqrt{2}}{2}R_{CE}\,2l + Y_D l = 0$$

$R_{CE} = -\,pl\,2\sqrt{2}$
$Y_D = 6pl$
$Y_A = -2pl$



- Riepilogo delle reazioni vincolari



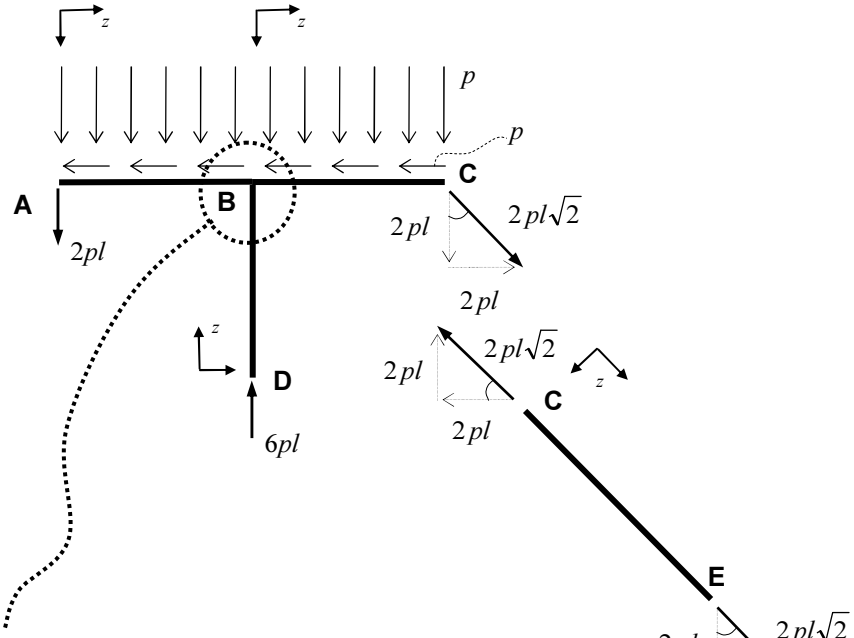
- ***Verifiche***

$\Sigma X = 0 \quad \Rightarrow \quad -2pl + \int_0^{2l} p\,dx = 0$ ✓

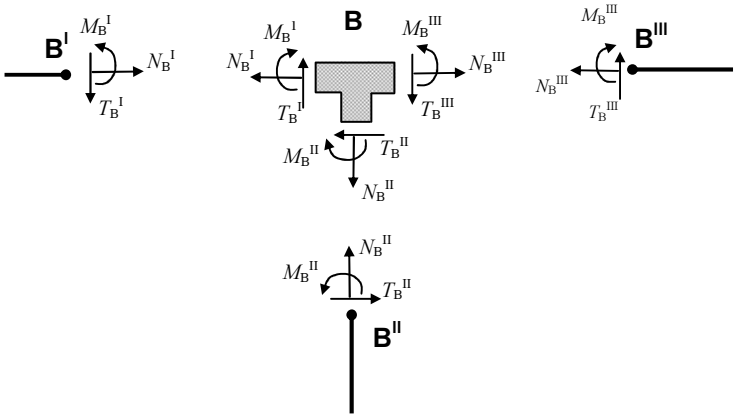
$\Sigma Y = 0 \quad \Rightarrow \quad -2pl + 6pl - 2pl - \int_0^{2l} p\,dx = 0$ ✓

$\Sigma M_D = 0 \quad \Rightarrow \quad 2pl^2 + 2pl^2 - 2pl^2 - \int_0^{2l} pl\,dx = 0$ ✓

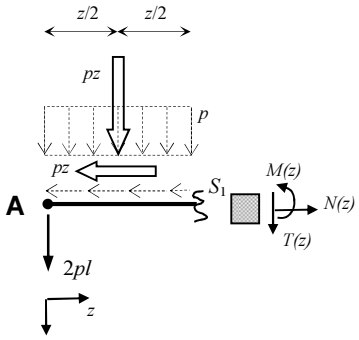
- Caratteristiche della Sollecitazione



- <u>NODO B</u>



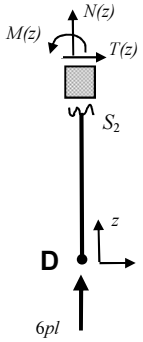
- *Tratto AB*



$N(z) = pz$
$T(z) = -2pl - pz$
$M(z) = -2pl\, z - (1/2)pz^2$

- $z = 0 \Rightarrow S_1 \equiv A$ $\quad N(0) = N_A = 0$, $T(0) = T_A = -2pl$, $M(0) = M_A = 0$
- $z = l \Rightarrow S_1 \equiv B^I$ $\quad N(l) = N_B{}^I = pl$, $T(l) = T_B{}^I = -3pl$, $M(l) = M_B{}^I = -(5/2)pl^2$

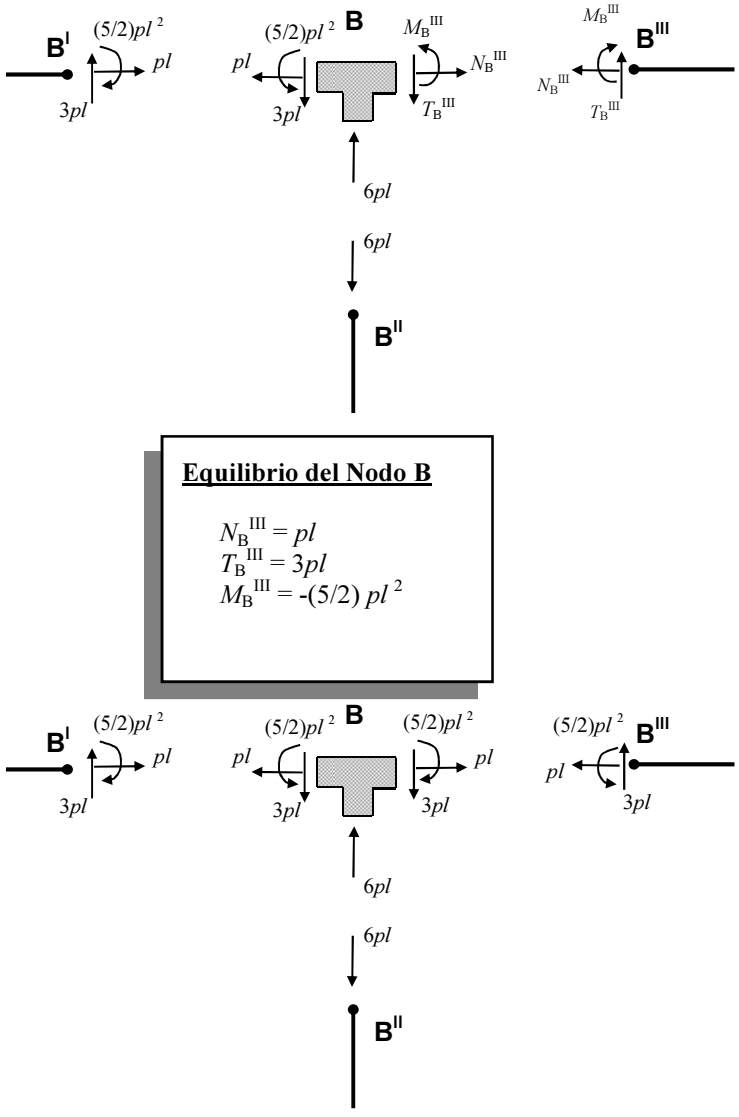
- *Tratto DB*



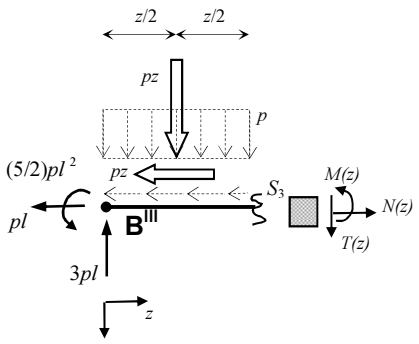
$N(z) = -6pl$
$T(z) = 0$
$M(z) = 0$

- $z = 0 \Rightarrow S_2 \equiv D$ $\quad N(0) = N_D = -6pl$, $T(0) = T_D = 0$, $M(0) = M_D = 0$
- $z = l \Rightarrow S_2 \equiv B^{II}$ $\quad N(l) = N_B{}^{II} = -6pl$, $T(l) = T_B{}^{II} = 0$, $M(l) = M_B{}^{II} = 0$

- *Nodo B*


B^I
(5/2)pl^2
pl
3pl
B
(5/2)pl^2
pl
3pl
M_B^III
N_B^III
T_B^III
B^III
M_B^III
N_B^III
T_B^III
6pl
6pl
B^II
Equilibrio del Nodo B
N_B^III = pl
T_B^III = 3pl
M_B^III = -(5/2) pl^2
B^I
(5/2)pl^2
pl
3pl
B
(5/2)pl^2
(5/2)pl^2
pl
pl
3pl
3pl
B^III
(5/2)pl^2
pl
3pl
6pl
6pl
B^II

- *Tratto BC*



$N(z) = pl + pz$
$T(z) = 3pl - pz$
$M(z) = -(5/2)pl^2 + 3pl\,z - (1/2)pz^2$

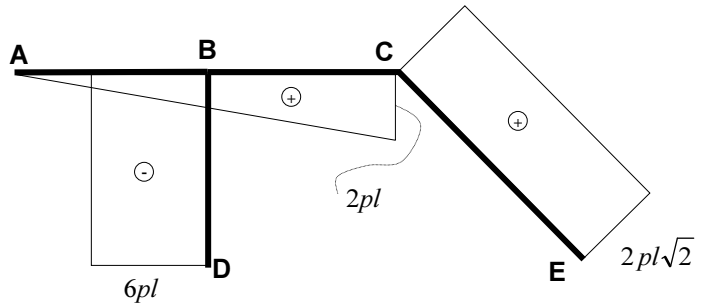
- $z = 0 \Rightarrow S_3 \equiv \mathrm{B}^{\mathrm{III}}$ $N(0) = N_{\mathrm{B}}^{\mathrm{III}} = -pl$
  $T(0) = T_{\mathrm{B}}^{\mathrm{III}} = 3pl$
  $M(0) = M_{\mathrm{B}}^{\mathrm{III}} = -(5/2)pl^2$

- $z = l \Rightarrow S_3 \equiv \mathrm{C}$ $N(l) = N_{\mathrm{C}} = 2pl$
  $T(l) = T_{\mathrm{C}} = 2pl$
  $M(l) = M_{\mathrm{C}} = 0$

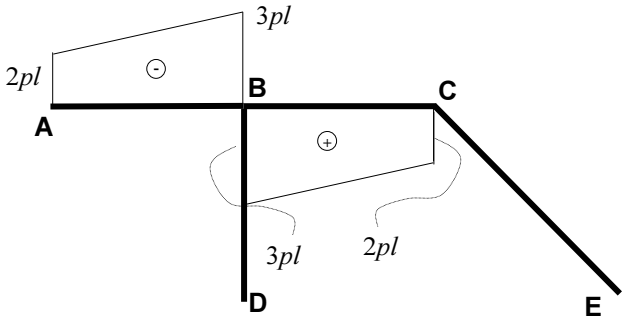
- *Tratto CE*

$N(z) = 2\,pl\sqrt{2}$
$T(z) = 0$
$M(z) = 0$

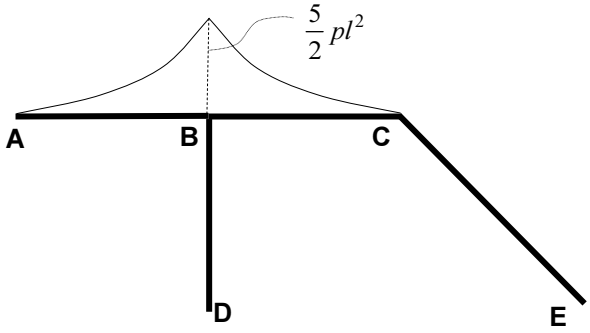
- Diagrammi delle caratteristiche della sollecitazione



N



T



M